Fondamenti della Meccanica Quantistica

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anno accademico 2025-2026
lezione 3

Programma indicativo del corso

Lezione 1

- Teorie pre-quantistiche
- Introduzione alla località, all’ontologia e alla misura
- Esempi quantistici: diffrazione, interferenza e stato entangled
- Il problema della misura

Lezione 2

- Il problema della località
- Il problema dell’ontologia
- L’interpretazione di Copenhagen

Lezione 3

- Dimostrazione sperimentale dei walker
- Le teorie dell’onda pilota del XX secolo: de Broglie e Bohm
- Dualità onda-particella in fisica classica: i walker
- Le nascenti teorie dell’onda pilota (lezione congiunta con A. Cirimele e F. Greco)

LEZIONE 3 - Teoria dell'onda pilota di Bohm e de Broglie, Walkers, Nuove teorie dell'onda pilota

16-12-2025

LA TEORIA AD ONDA PILOTA DI BOHM

[Bell - - p. 178]

S3

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi$$

$$p = \frac{h}{\lambda} = \hbar k$$ formula di de Broglie

Nel caso di un'onda piana $\Psi \sim e^{ikx}$ la velocità della particella si può scrivere

$$v = \frac{p}{m} = \frac{\hbar k}{m}$$

Nel caso di un'onda qualsiasi scriviamo

$$\Psi(x,t) = \underset{\substack{\uparrow \\ \text{ampiezza}}}{R(x,t)}\, e^{i \underset{\substack{\uparrow \\ \text{fase}}}{S(x,t)}}$$

e $v = \frac{\hbar}{m}\frac{\partial S}{\partial x}$

$S(x,t) \sim kx$ per un'onda piana.

Poi ci serve un'equazione che descrive il moto della particella.

POSTULATO:
EQUAZIONE DEL MOTO DELLA PARTICELLA

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\frac{\partial S(x,t)}{\partial x}\bigg|_{x=X(t)}$$

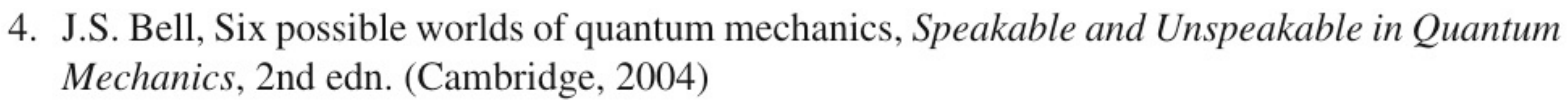
Notare che $S(x,t)$ in generale è complessa.

L'ultima equazione può essere riscritta nel modo seguente

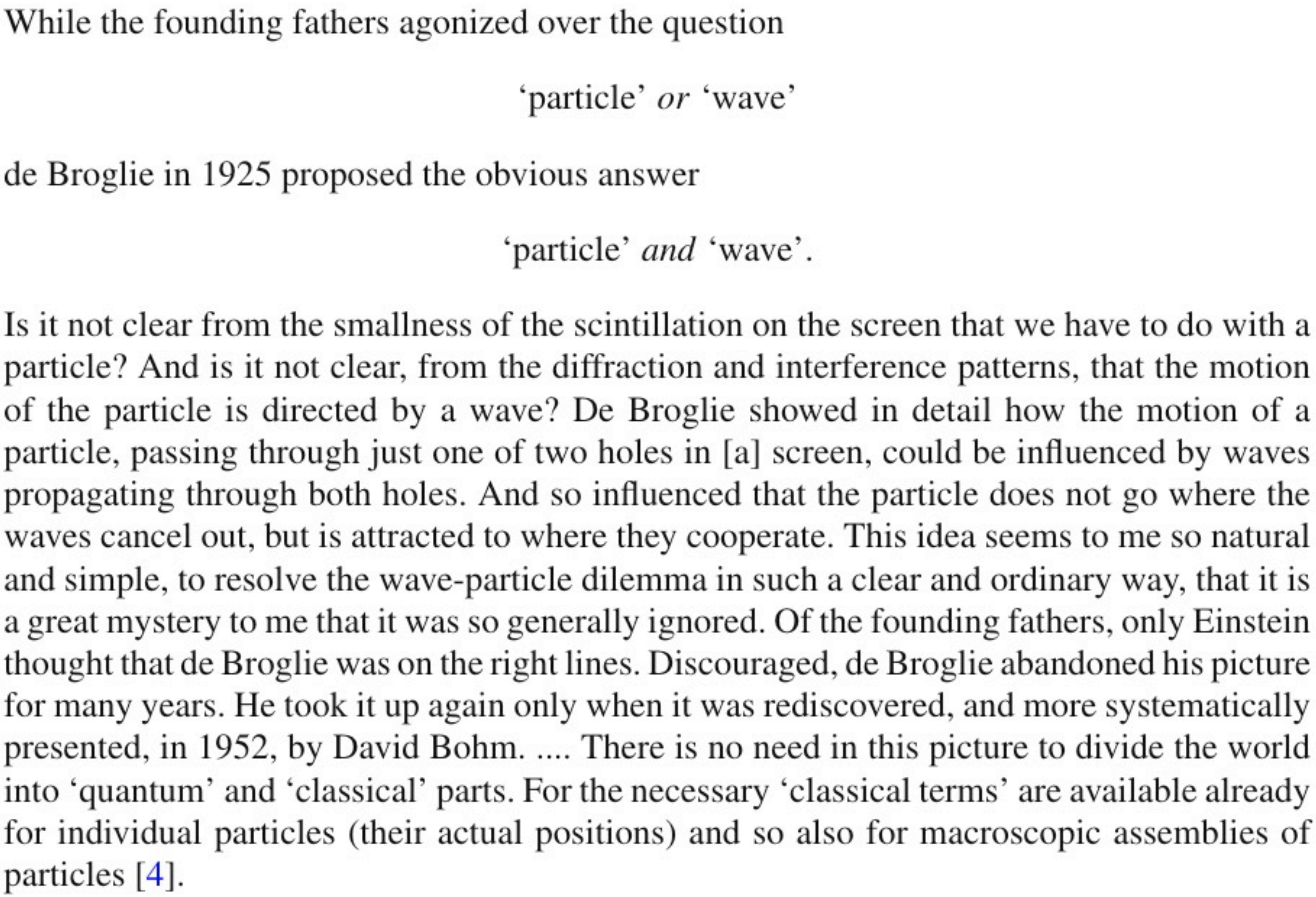
$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\,\mathrm{Im}\left[\frac{\frac{\partial \Psi}{\partial x}}{\Psi}\right]\bigg|_{x=X(t)}$$

La teoria ad onda pilota di Bohm consiste in questo.

Sviluppiamone un aspetto.



4. J.S. Bell, Six possible worlds of quantum mechanics, *Speakable and Unspeakable in Quantum Mechanics*, 2nd edn. (Cambridge, 2004)



While the founding fathers agonized over the question

‘particle’ *or* ‘wave’

de Broglie in 1925 proposed the obvious answer

‘particle’ *and* ‘wave’.

Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in [a] screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored. Of the founding fathers, only Einstein thought that de Broglie was on the right lines. Discouraged, de Broglie abandoned his picture for many years. He took it up again only when it was rediscovered, and more systematically presented, in 1952, by David Bohm. .... There is no need in this picture to divide the world into ‘quantum’ and ‘classical’ parts. For the necessary ‘classical terms’ are available already for individual particles (their actual positions) and so also for macroscopic assemblies of particles [4].

From “Foundations of Quantum Mechanics” by Travis Norsen, Springer 2017.

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x,t)\,\Psi$$

complessa coniugata
$$-i\hbar \frac{\partial \Psi^*}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi^*}{\partial x^2} + V(x,t)\,\Psi^* \qquad \text{con } V(x,t) \text{ assunto reale}$$

Moltiplichiamo la prima per Ψ^*, la seconda per Ψ e sottraiamo la seconda dalla prima:

$$i\hbar\left[\Psi^*\frac{\partial \Psi}{\partial t} + \Psi\frac{\partial \Psi^*}{\partial t}\right] = -\frac{\hbar^2}{2m}\left[\Psi^*\frac{\partial^2 \Psi}{\partial x^2} + \Psi\frac{\partial^2 \Psi^*}{\partial x^2}\right]$$

$$\Rightarrow \quad \frac{\partial}{\partial t}|\Psi|^2 = -\frac{\partial}{\partial x}\left[\frac{i\hbar}{2m}\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right)\right]$$

questa ha la forma di un'equazione di continuità in cui $\rho = |\Psi|^2$:

$$\frac{\partial}{\partial t}\rho = -\frac{\partial}{\partial x}\vec{j}$$

o in 3 dimensioni:

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}\cdot\vec{j}$$

Questa equazione può essere intesa come l'equazione per la conservazione locale della probabilità, in cui $\vec{j}$ è la corrente di probabilità.

A questo punto, nel contesto della meccanica Bohmiana, ci aspetteremmo che la velocità della particella sia:

$$v = \frac{j}{\rho} = \frac{i\hbar}{2m}\,\frac{\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi}{\Psi^*\Psi} =$$

$$= \ldots = \frac{\hbar}{m}\,\mathrm{Im}\left[\frac{\left(\frac{\partial \Psi}{\partial x}\right)}{\Psi}\right] \qquad \checkmark$$

in accordo con l'equazione del moto postulata.

Qui la regola di Born può essere derivata anziché postulata.

Particella in una cavità

[S4-5 - p. 183, 184, 185]

Con le "buone" condizioni iniziali, la teoria di Bohm è perfettamente in accordo con le predizioni della formulazione di Copenhagen.

Diffrazione e interferenza

[S6-7 - p. 187, 188]

Misura

La teoria ad onda pilota di Bohm ci permette di non dividere il mondo in sistemi quantici e sistemi classici per capire cosa vuol dire misurare. Questa teoria non soffre del problema della misura.

Riprendiamo la nostra hamiltoniana d'interazione tra il sistema di misura e la particella in cavità discussa in precedenza:

$$\hat{H}_{int} = \lambda \hat{H}_x \hat{p}_y$$

S8

con condizione iniziale

$$\Psi(x, y, 0) = \varphi_n(x)\,\phi(y)$$

L'equazione di Schrödinger ci dà l'evoluzione temporale

$$\Psi(x, y, t) = \varphi_n(x)\,\phi(y - \lambda E_n t)$$

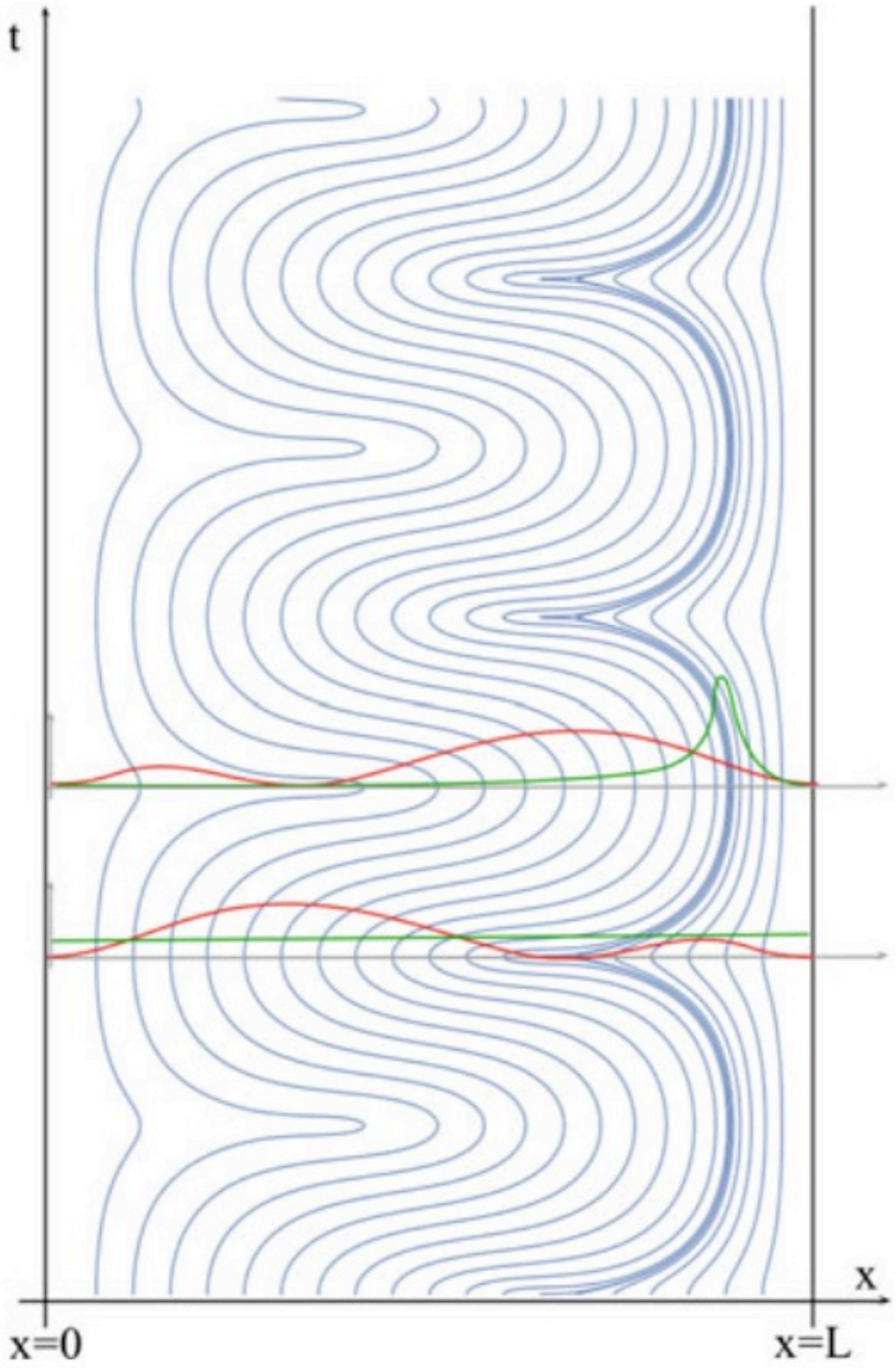
Ricordiamo che il problema nasceva quando la particella si trovava inizialmente in una sovrapposizione di stati

$$\Psi(x, y, 0) = \left[\sum_i c_i \varphi_i(x) \right] \phi(y)$$

Buca di potenziale infinito

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left[\psi_1(x)e^{-iE_1t/\hbar} + \psi_2(x)e^{-iE_2t/\hbar}\right]$$
$$= \frac{1}{\sqrt{L}}\left[\sin(\pi x/L)e^{-i\omega_1 t} + \sin(2\pi x/L)e^{-i\omega_2 t}\right].$$

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\mathrm{Im}\left[\frac{\frac{\pi}{L}\cos\left(\frac{\pi x}{L}\right)e^{-i\omega_1 t} + \frac{2\pi}{L}\cos\left(\frac{2\pi x}{L}\right)e^{-i\omega_2 t}}{\sin\left(\frac{\pi x}{L}\right)e^{-i\omega_1 t} + \sin\left(\frac{2\pi x}{L}\right)e^{-i\omega_2 t}}\right]_{x=X(t)}$$



$$\Psi(x,t)$$

$$P(x,t)$$

From “Foundations of Quantum Mechanics” by Travis Norsen, Springer 2017.

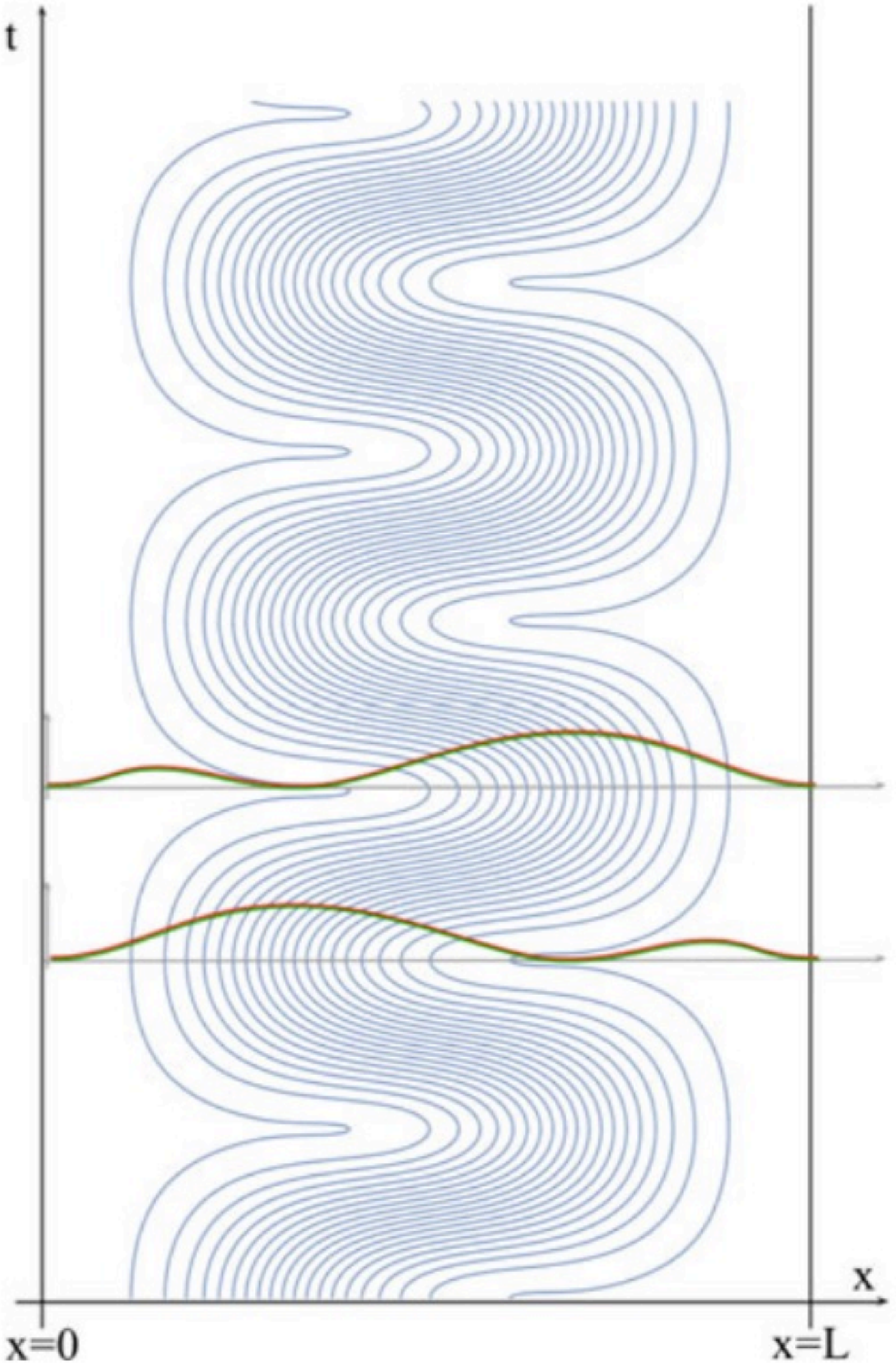
Buca di potenziale infinito

$$\begin{aligned}\Psi(x,t) &= \frac{1}{\sqrt{2}}\left[\psi_1(x)e^{-iE_1t/\hbar} + \psi_2(x)e^{-iE_2t/\hbar}\right] \\ &= \frac{1}{\sqrt{L}}\left[\sin(\pi x/L)e^{-i\omega_1 t} + \sin(2\pi x/L)e^{-i\omega_2 t}\right].\end{aligned}$$

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\mathrm{Im}\left[\frac{\frac{\pi}{L}\cos\left(\frac{\pi x}{L}\right)e^{-i\omega_1 t} + \frac{2\pi}{L}\cos\left(\frac{2\pi x}{L}\right)e^{-i\omega_2 t}}{\sin\left(\frac{\pi x}{L}\right)e^{-i\omega_1 t} + \sin\left(\frac{2\pi x}{L}\right)e^{-i\omega_2 t}}\right]_{x=}$$

$\Psi(x,t)$

$P(x,t)$



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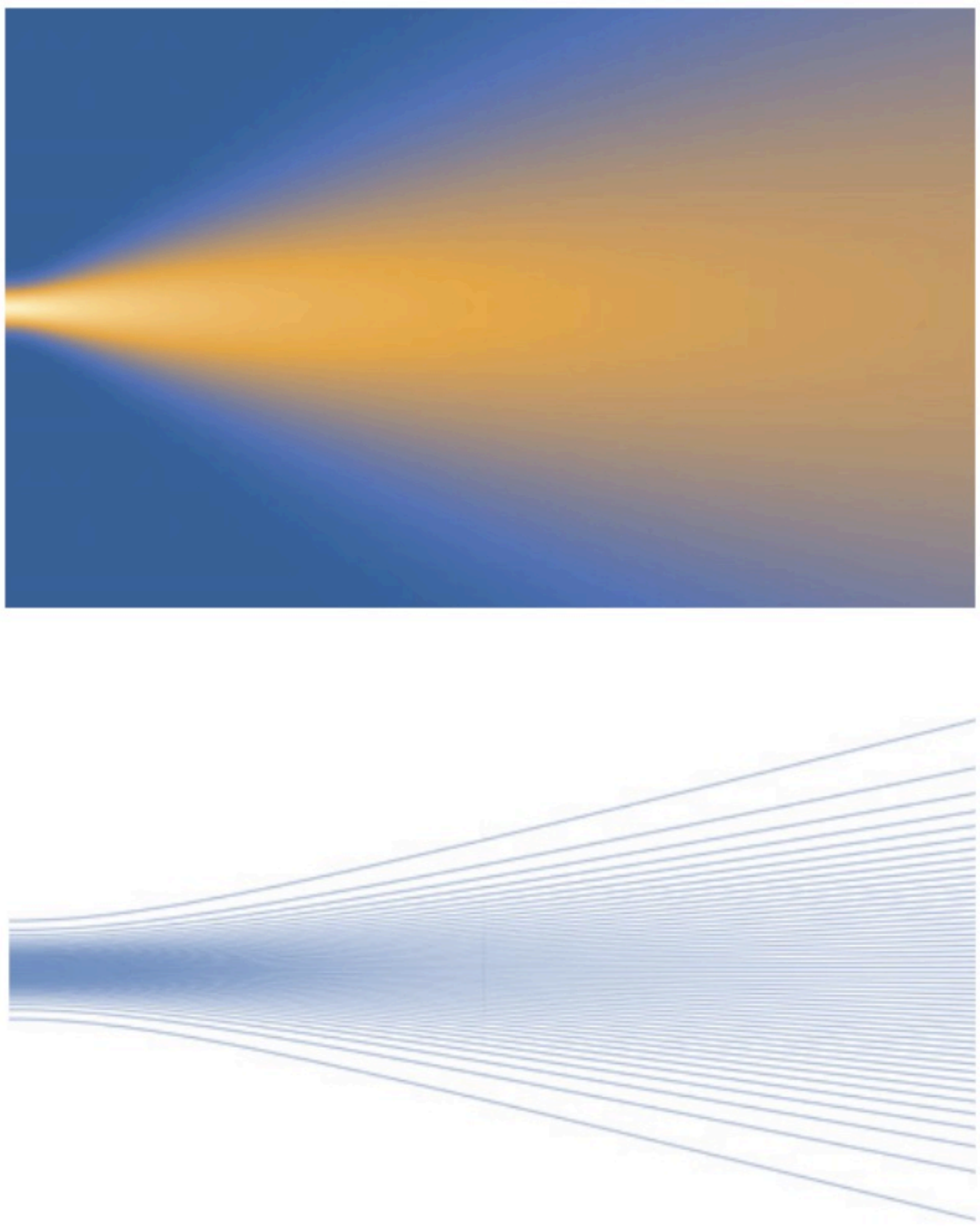
Diffrazione



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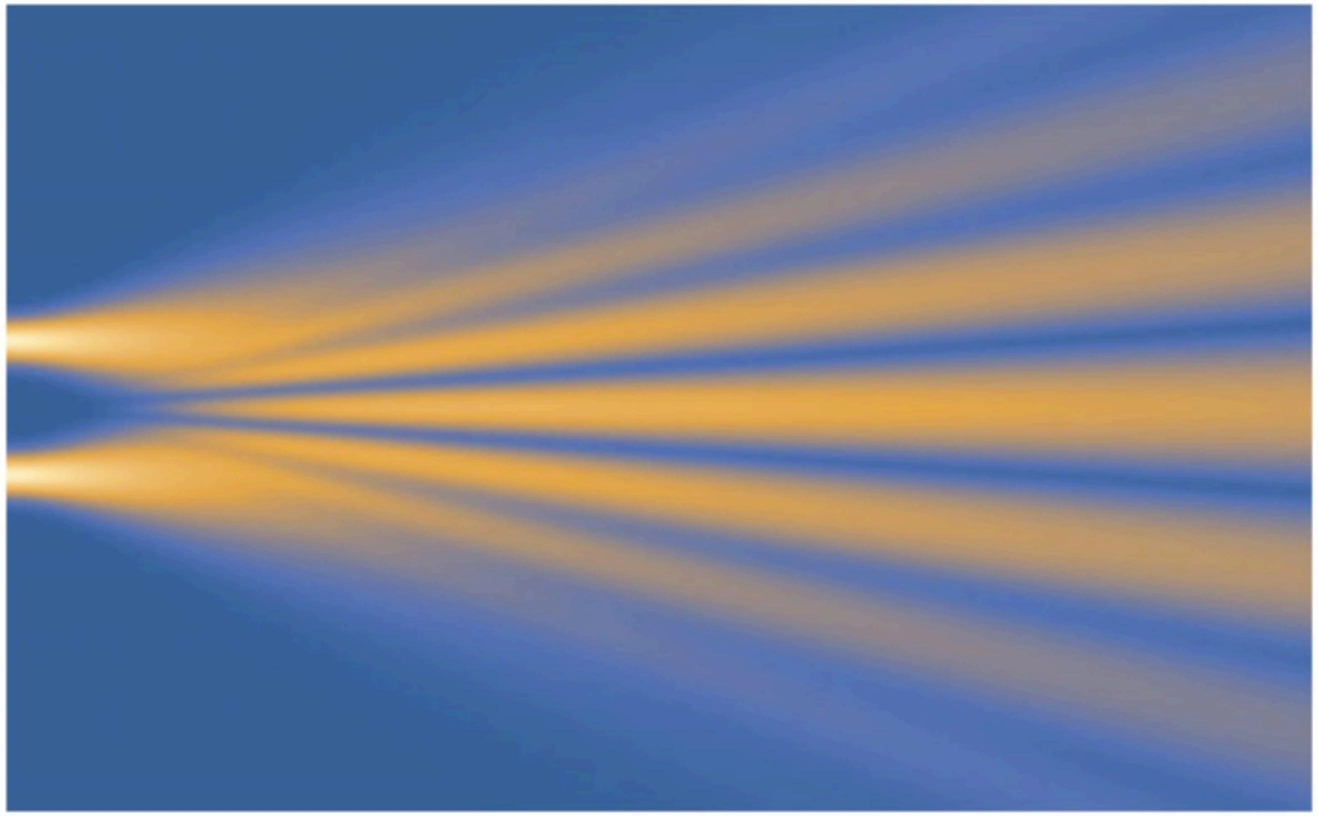
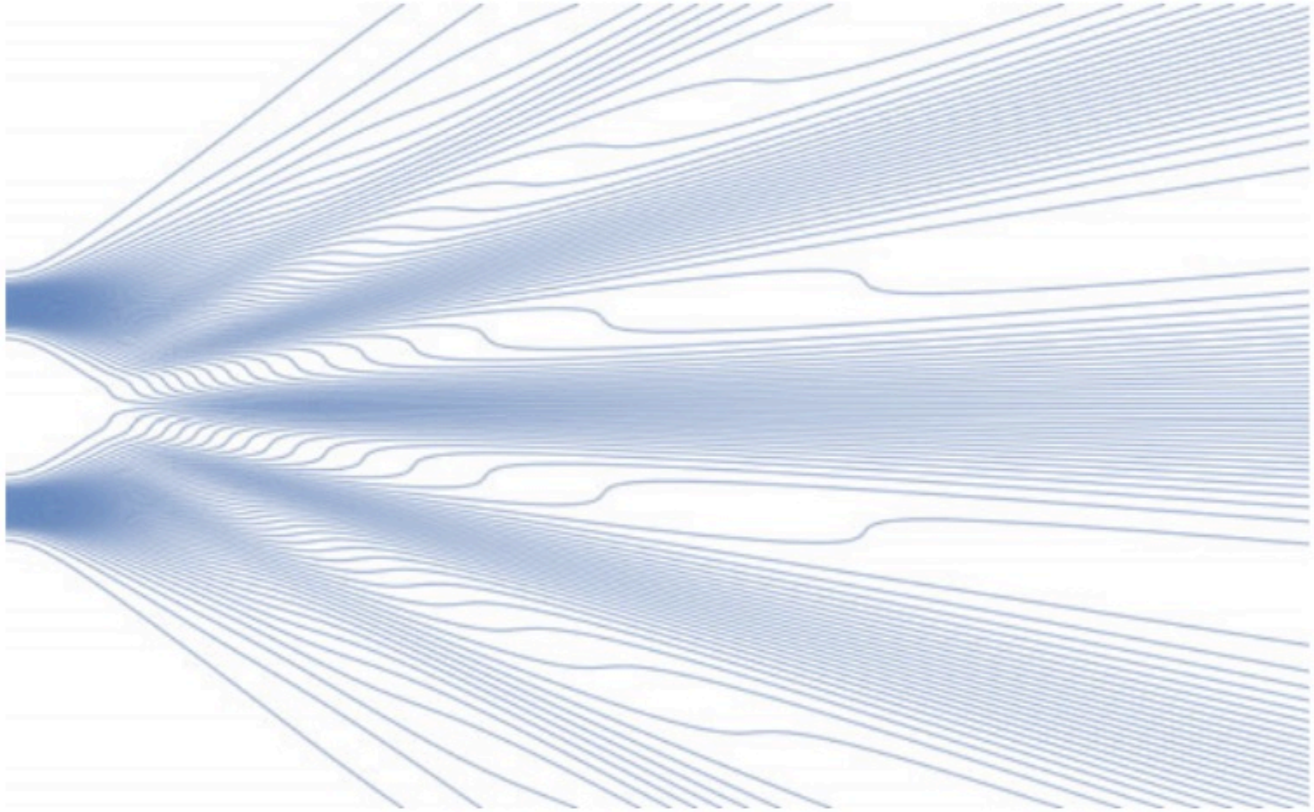
Interferenza





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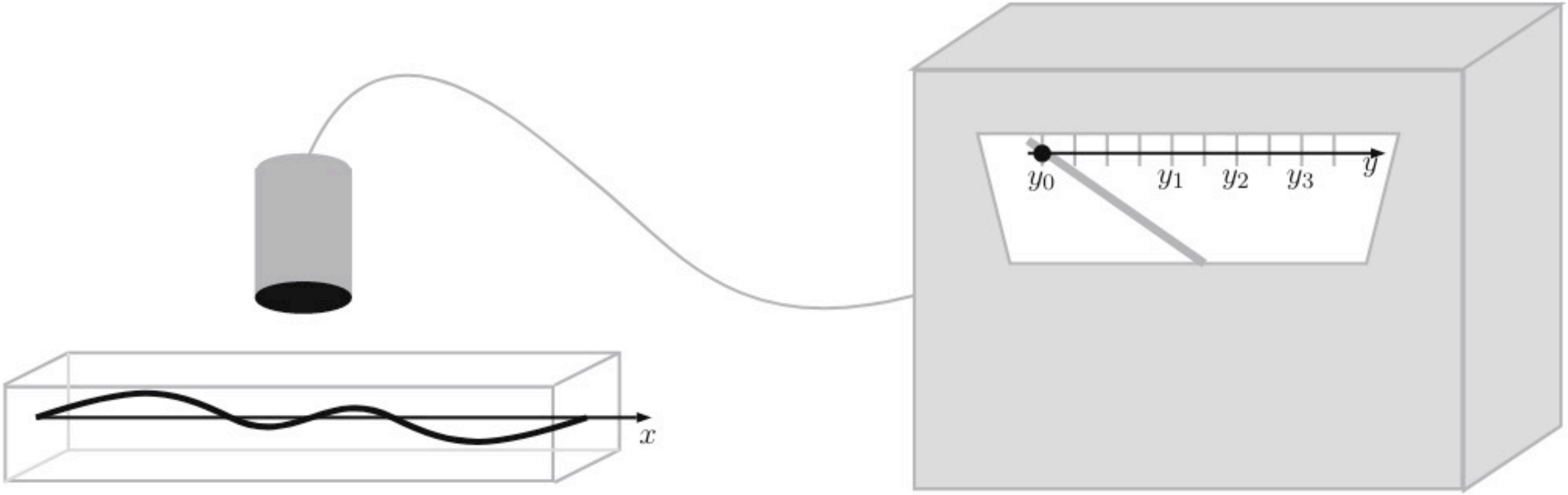


Fig. 3.1 The quantum particle-in-a-box (whose spatial degree of freedom is called x) is shown on the *left*; the curve is meant to indicate its wave function (though one should be careful not to take this picture too literally!). Then there is an energy-measurement device which will perform the measurement. The device has a macroscopic pointer, which we can idealize as a single, very heavy particle with horizontal coordinate y. Prior to the measurement-interaction, the pointer is sitting in its "ready" position (y_0); after the measurement interaction, the pointer will move to a new position which indicates the outcome of the measurement: y_1 will mean that the energy of the particle is E_1, etc

From "Foundations of Quantum Mechanics" by Travis Norsen, Springer 2017.

poiché si ottenere uno stato entangled tra la particella nella scatola e l'apparato di misura:

$$\Psi(x,y,t) = \sum_i c_i \, \psi_i(x) \, \phi(y - \lambda E_i t)$$

in cui né la particella è ad un livello di energia definito né il puntatore è in una posizione definita, in disaccordo con le osservazioni sperimentali.

Secondo la meccanica Bohmiana, la particella nella scatola si muove secondo

$$\frac{d\,X(t)}{dt} = \frac{j_x}{|\Psi|^2}$$

mentre la posizione dell'articella segue

$$\frac{d\,Y(t)}{dt} = \frac{j_y}{|\Psi|^2}$$

Il problema è risolto: S9

OSSERVAZIONI IMPORTANTI

1) La teoria dell'onda pilota di Bohm produce LE STESSE predizioni statistiche della meccanica quantistica ordinaria, sebbene la teoria sia completamente deterministica.

2) Non serve il postulato del collasso della funzione d'onda, poiché il sistema obbedisce SEMPRE all'equazione di Schrödinger.

3) La casualità è presente solo nelle condizioni iniziali, come in meccanica statistica classica.

4) Ogni particella dell'universo esiste ed ha posizione e momento definiti, ma queste non sono tutte accessibili a noi (il principio di indeterminazione di Heisenberg quindi riguarda ciò che ESISTONO).

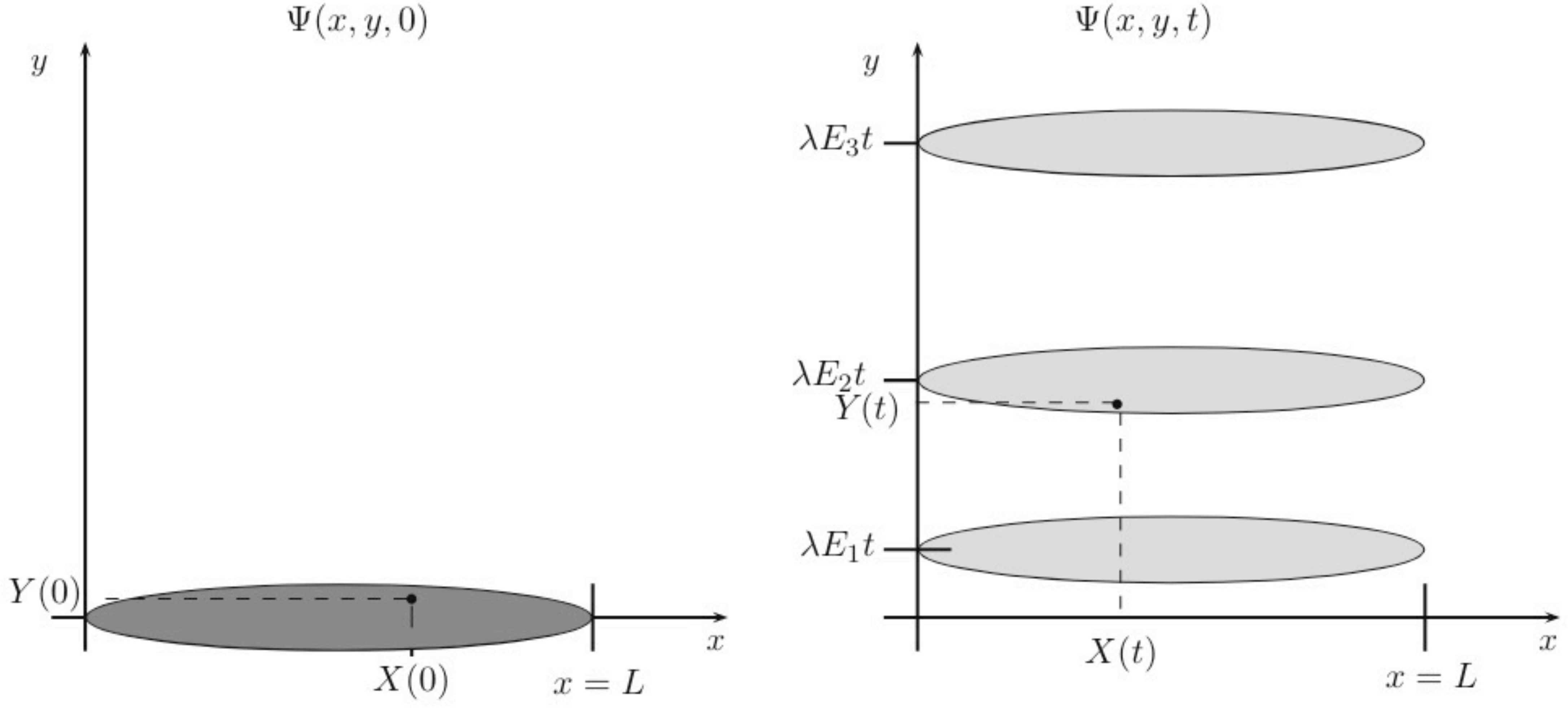


Fig. 7.5 The graph on the *left highlights* (in *dark gray*) the region of the two-dimensional configuration space where $\Psi(x, y, 0)$ has support. Later, at time t, the wave function has split apart into several non-overlapping "islands". This is depicted in the graph on the *right*. The simultaneous presence of all these islands constitutes, for orthodox quantum mechanics, the measurement problem. But for the pilot-wave theory, the actually-realized outcome of the measurement is not to be found in the wave function, but rather in the final position of the pointer. And this, in the pilot-wave theory, will be some one (random but perfectly definite) value, indicated here by the vertical position $Y(t)$ of the *dot* which represents the actual configuration point (X, Y). The indicated $Y(t)$ is in the support of the $n = 2$ branch of the wave function – i.e., $Y(t)$ is approximately $\lambda E_2 t$ – so we would say in this case that the energy measurement had the outcome $E = E_2$. Note that the outcome might have been different had the (random) initial positions $X(0)$ and $Y(0)$ been different

From "Foundations of Quantum Mechanics" by Travis Norsen, Springer 2017.

Sebbene ci sia, in un certo senso, una sola grande funzione d'onda che descrive l'universo, la teoria ad onda pilota permette di definire la funzione d'onda per un sotto-sistema.

Riprendiamo il caso precedente

$$\Psi(x,y,t) = \sum_i c_i \varphi_i(x)\, \phi(y - \lambda E_i t)$$

calcolando il valore della funzione in $y = Y(t)$, cioè nella posizione del puntatore, si ottiene

$$\chi(x,t) \sim \sum_i c_i \varphi_i(x)\, \phi(Y(t) - \lambda E_i t)$$

A $t=0$, cioè prima dell'inizio della misura, la funzione d'onda è

$$\chi(x,0) \sim \sum_i c_i \varphi_i(x)\, \underbrace{\phi(Y(0))}_{\text{costante}} \sim \sum_i c_i \varphi_i(x)$$

Alla fine della misura $Y(t) \approx \lambda E_n t$ per un n particolare che è il risultato reale dell'esperimento.

Poiché ϕ è una specie di pacchetto gaussiano piuttosto stretto, $\phi(Y(t) - \lambda E_i t) \approx 0 \quad \forall i \neq n$. Quindi

$$\chi(x,t) \sim \sum_i c_i \varphi_i(x)\, \phi(Y(t) - \lambda E_i t) \approx c_n \varphi_n(x)\, \phi(Y(t) - \lambda E_n t)$$

$\Rightarrow \chi(x,t) = \varphi_n(x)$ poiché gli altri due fattori sono costanti che non dipendono da x.

Questo "spiega" il collasso della funzione d'onda.

CONTESTUALITÀ

Notiamo che, sebbene la misura restituisca un risultato definito, questo non vuol dire che la particella avesse un'energia definita prima della misura.
Quindi, anche nella teoria dell'onda pilota di Bohm la misura non è semplicemente una rivelazione passiva di un valore pre-esistente.

Consideriamo un elettrone in un potenziale $V(x) = \frac{1}{2} m \omega^2 x^2$ la soluzione dell'equazione di Schrödinger di minore energia indipendente dal tempo è una gaussiana

$$\psi(x) = N e^{-\frac{x^2}{4\sigma^2}} \qquad \text{con} \quad \sigma^2 = \frac{\hbar}{2m\omega}$$

$$E = \frac{1}{2}\hbar\omega$$

$$\Rightarrow \quad \psi(x,t) = N e^{-\frac{x^2}{4\sigma^2}} e^{-i\frac{Et}{\hbar}}$$

Poiché $S(x,t)$ dipende solo da t, $v = 0$.
Se assumiamo che la quantità di moto della particella è data dalla formula $p = m \frac{dX(t)}{dt}$, allora $p = 0$.

Ma questo è un problema perché questo risultato è assolutamente poco probabile. Infatti se scriviamo

$$\psi(x) = \int \phi(k) \frac{e^{ikx}}{\sqrt{2\pi}} dk \quad , \quad \text{con} \quad \phi(k) = \sqrt{2} N \sigma e^{-k^2\sigma^2}$$

$$\Rightarrow \quad P(p)\,dp = P(k)\,dk = |\phi(k)|^2 dk = 2N^2\sigma^2 e^{-2k^2\sigma^2} dk =$$
$$= \frac{2N^2\sigma^2}{\hbar} e^{-2p^2\sigma^2/\hbar^2} dp$$

dove abbiamo usato $p = \hbar k$.

Questo ci dice che, se la teoria ad onda pilota oltre riprodurre le predizioni statistiche della meccanica quantistica ordinaria, allora non può essere che una misura della quantità di moto riveli semplicemente un valore di quantità di moto pre-esistente.

In questa teoria ad onda pilota la quantità di moto è misurata tramite una procedura denominata "tempo di volo". L'idea è di "spegnere" temporaneamente il potenziale $V(x)$, lasciare che la particella si sposti liberamente per un tempo lungo, osservare a un certo punto la sua posizione e dedurre la quantità di moto che le avrebbe permesso di arrivare lì.

Nell'esempio del potenziale armonico, questo consisterebbe nel considerare un pacchetto gaussiano che si sparpaglia

$$\Psi(x,0) = N e^{-x^2/4\sigma^2} \longrightarrow \Psi(x,t) = N(t)\, e^{-x^2/4\left(\sigma^2 + \frac{i\hbar t}{2m}\right)}$$

Riscrivendola in forma polare troviamo

$$S(x,t) = \frac{x^2 \hbar t}{8m\left(\sigma^4 + \hbar^2 t^2/4m^2\right)}$$

Usando l'equazione del moto otteniamo

$$\frac{dX(t)}{dt} = X(t) \frac{t}{t^2 + \frac{4m^2\sigma^4}{\hbar^2}}$$

che ha soluzione $X(t) = X_0 \left(1 + \frac{t^2}{4m^2\sigma^4/\hbar^2}\right)^{1/2}$

Per un tempo lungo queste si può approssimare

$$X(t) \approx X_0 \frac{\hbar t}{2 m \sigma^2}$$

Se al tempo t la particella viene osservata in X(t) ne deduciamo che la sua velocità è stata $v = \frac{X(t)}{t}$ per cui la quantità di moto

$$p = mv = \frac{X_0 \hbar}{2\sigma^2}$$

Osserviamo che il risultato della misura, anche qui in qualche modo, viene fuori intervenendo sul sistema, ma il modo in cui avviene (a differenza della MQ ordinaria) è piuttosto chiaro e non richiede l'introduzione di ulteriori postulati.

Questo metodo di misura della quantità di moto è in accordo con la predizione statistica. Infatti, assumendo che la probabilità di misurare la quantità di moto nell'intervallo compreso tra p e p + dp sia uguale alla probabilità che la misura di posizione iniziale fosse nell'intervallo che dà quel risultato della quantità di moto:

$$P(p)\,dp = P(X_0)\,dX_0 = |\psi(X_0, 0)|^2 \, dX_0 =$$

$$= N^2 e^{-\frac{X_0^2}{2\sigma^2}} \, dX_0 = \frac{2N^2\sigma^2}{\hbar} e^{-\frac{2p^2\sigma^2}{\hbar^2}} \, dp$$

che corrisponde a quella predetta dalla regola di Born generalizzata nella meccanica quantistica ordinaria.

L'esempio descritto mostra come, per quantità misurabili diverse dalla posizione (come quantità di moto ed energia, per esempio), questa teoria ad onde pilota non riveli semplicemente valori pre-esistenti. Infatti prima della misura avevamo p=0.

Inoltre, il risultato della misura dipende non solo dallo stato iniziale del sistema, ma anche dal modo in cui la misura viene effettuata.

In questo senso si dice che queste proprietà (quantità di moto, energia ...) sono CONTESTUALI.

Se per "MISURA" s'intende il rivelare qualcosa che era già definito, allora per questa teoria ad onde pilota quelle della posizione sono misure, le altre no.

Si potrebbe usare la parola più generale "esperimento" anziché "misura".

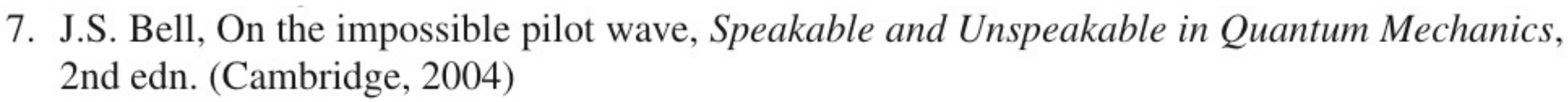
Variabili nascoste

Nel suo libro "Fondamenti matematici della meccanica quantistica" John von Neumann presentò una dimostrazione per cui non potevano esistere teorie a variabili nascoste che risultassero equivalenti alla meccanica quantistica.

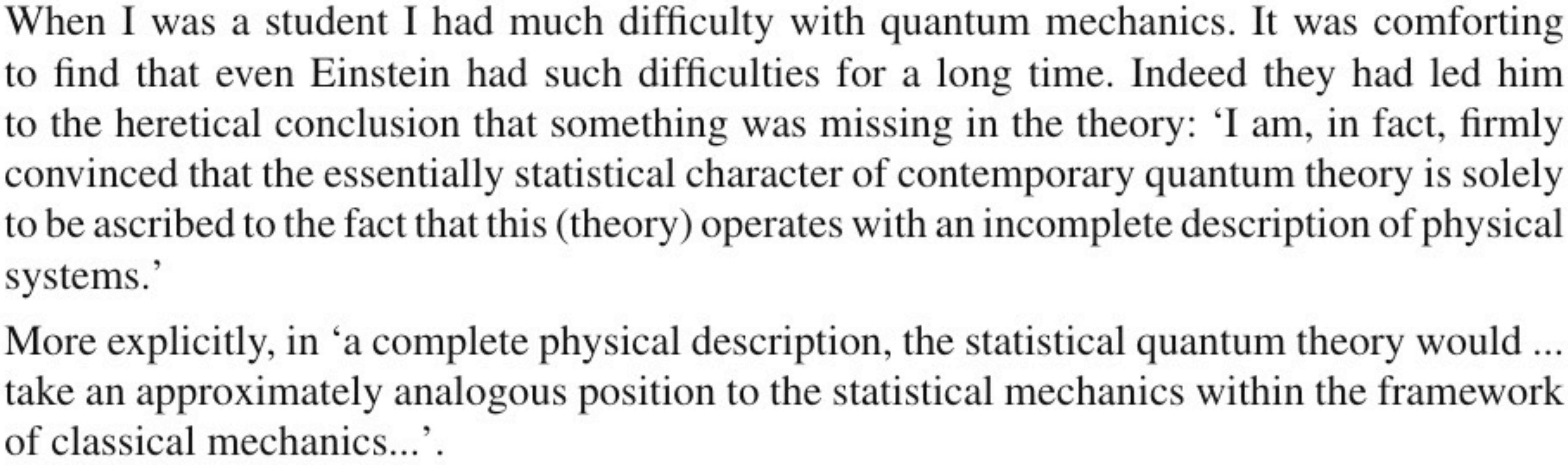
La validità della dimostrazione fu messa in dubbio da Grete Hermann (matematica e filosofa tedesca) tre anni dopo, che fu ignorata.

Nel 1966 Bell mostrò la fallacia della dimostrazione di Von Neumann nell' avere scelto ipotesi troppo restrittive.

[Bell - S10-11-12 - p. 199]



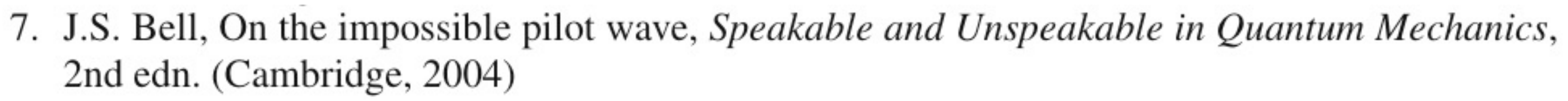
7. J.S. Bell, On the impossible pilot wave, *Speakable and Unspeakable in Quantum Mechanics*, 2nd edn. (Cambridge, 2004)



When I was a student I had much difficulty with quantum mechanics. It was comforting to find that even Einstein had such difficulties for a long time. Indeed they had led him to the heretical conclusion that something was missing in the theory: ‘I am, in fact, firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this (theory) operates with an incomplete description of physical systems.’

More explicitly, in ‘a complete physical description, the statistical quantum theory would ... take an approximately analogous position to the statistical mechanics within the framework of classical mechanics...’.

From “Foundations of Quantum Mechanics” by Travis Norsen, Springer 2017.



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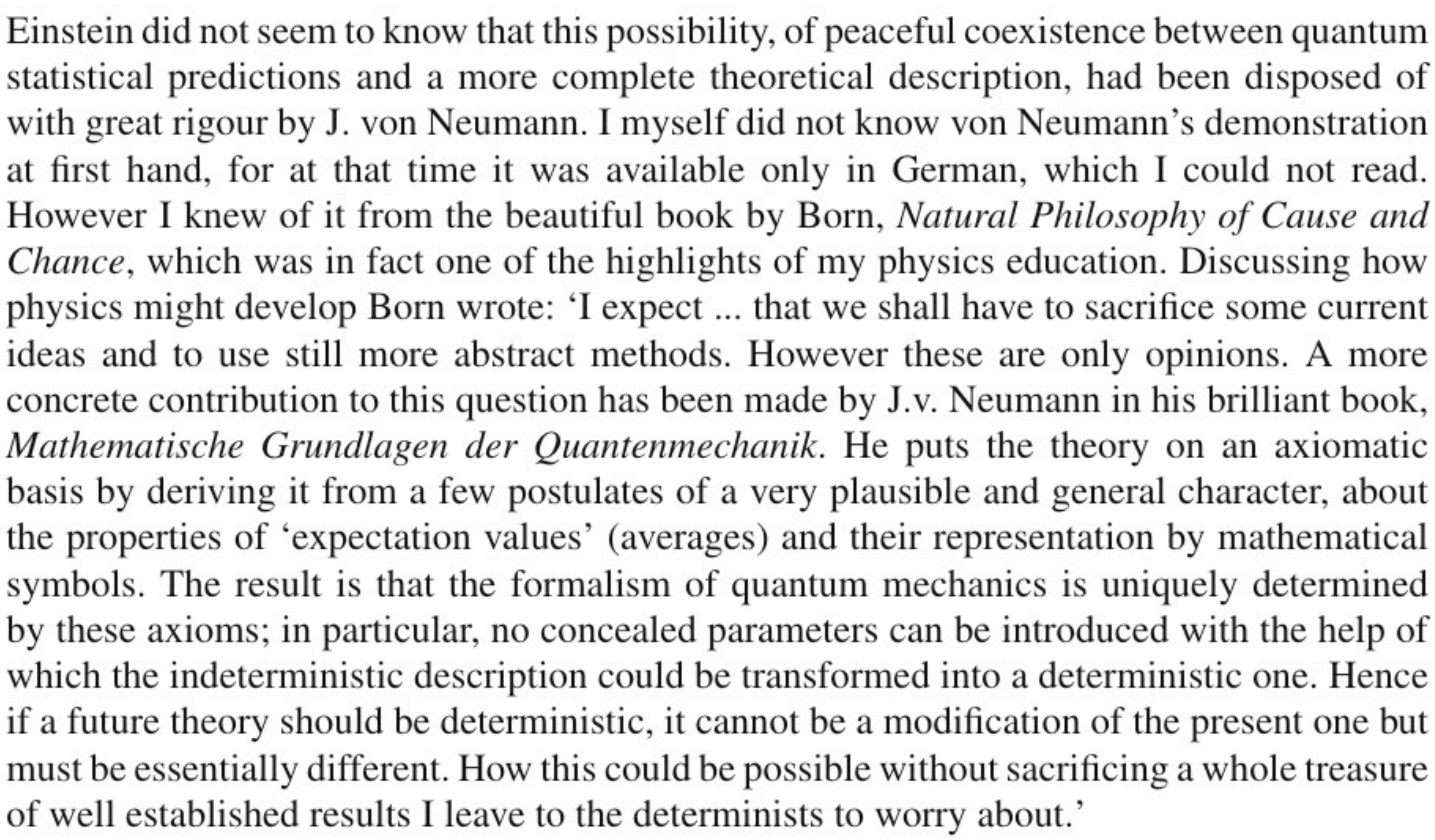

Einstein did not seem to know that this possibility, of peaceful coexistence between quantum statistical predictions and a more complete theoretical description, had been disposed of with great rigour by J. von Neumann. I myself did not know von Neumann's demonstration at first hand, for at that time it was available only in German, which I could not read. However I knew of it from the beautiful book by Born, *Natural Philosophy of Cause and Chance*, which was in fact one of the highlights of my physics education. Discussing how physics might develop Born wrote: 'I expect ... that we shall have to sacrifice some current ideas and to use still more abstract methods. However these are only opinions. A more concrete contribution to this question has been made by J.v. Neumann in his brilliant book, *Mathematische Grundlagen der Quantenmechanik*. He puts the theory on an axiomatic basis by deriving it from a few postulates of a very plausible and general character, about the properties of 'expectation values' (averages) and their representation by mathematical symbols. The result is that the formalism of quantum mechanics is uniquely determined by these axioms; in particular, no concealed parameters can be introduced with the help of which the indeterministic description could be transformed into a deterministic one. Hence if a future theory should be deterministic, it cannot be a modification of the present one but must be essentially different. How this could be possible without sacrificing a whole treasure of well established results I leave to the determinists to worry about.'

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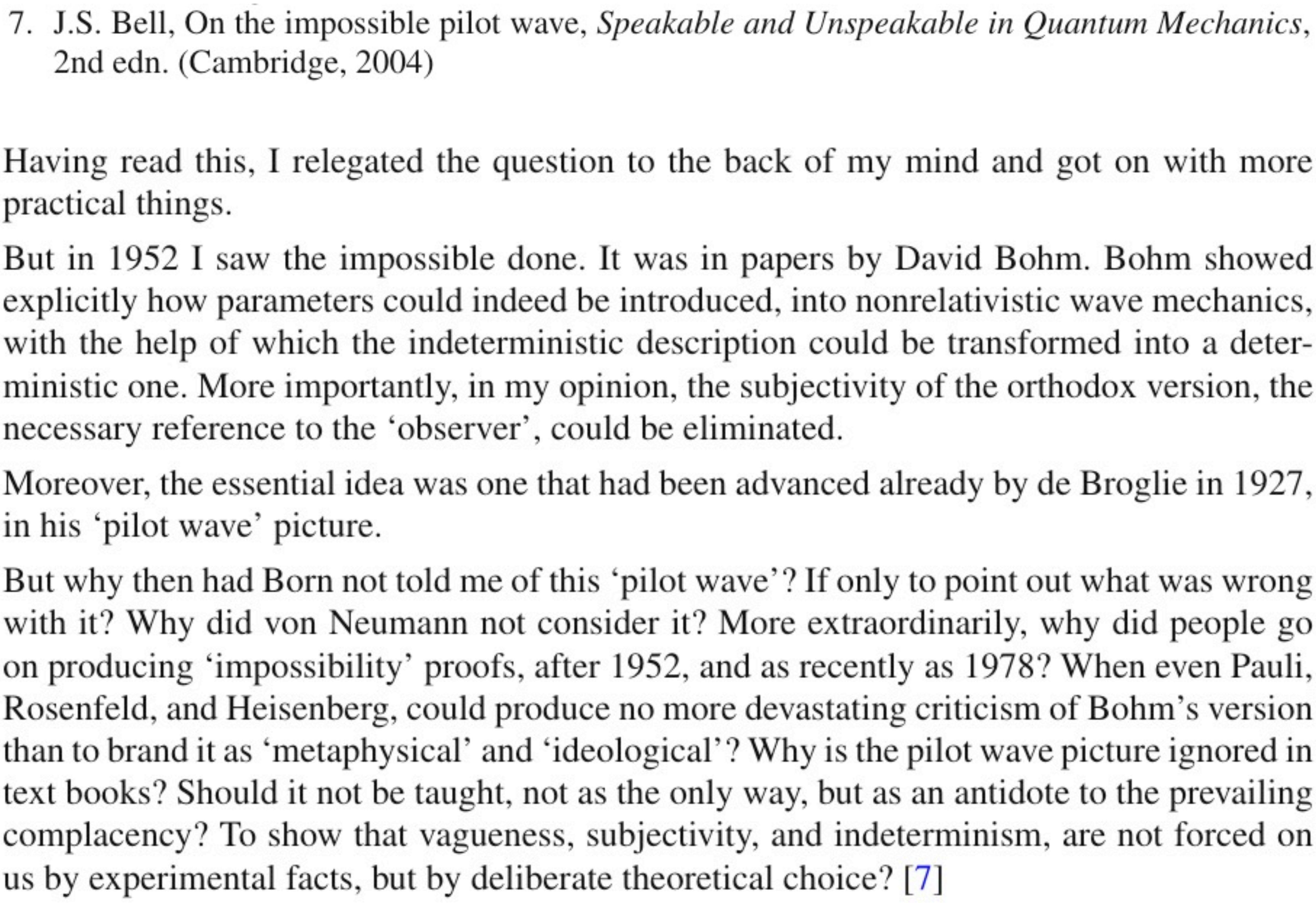

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Having read this, I relegated the question to the back of my mind and got on with more practical things.

But in 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the 'observer', could be eliminated.

Moreover, the essential idea was one that had been advanced already by de Broglie in 1927, in his 'pilot wave' picture.

But why then had Born not told me of this 'pilot wave'? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing 'impossibility' proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg, could produce no more devastating criticism of Bohm's version than to brand it as 'metaphysical' and 'ideological'? Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice? [7]

From "Foundations of Quantum Mechanics" by Travis Norsen, Springer 2017.

Molte particelle e località

Problema: la teoria ad onda pilota di Bohm non risolve il problema della località, anzi lo rende più esplicito ed eclatante.

Per esempio consideriamo un sistema di due particelle con funzione d'onda $\Psi(x_1, x_2, t)$. La velocità della prima particella al tempo t è data da

$$v_1(t) = \frac{d X_1(t)}{dt} = \frac{\hbar}{m_1} \mathrm{Im} \left[\frac{\frac{\partial \Psi(x_1, X_2(t), t)}{\partial x_1}}{\Psi(x_1, X_2(t), t)} \right]_{x_1 = X_1(t)}$$

Il moto della particella 1 dipende istantaneamente dalla posizione della particella 2!

Se assumere la relatività consiste nel vietare influenze causali più veloci della luce allora la teoria ad onda pilota contraddice la relatività.

Reazioni dei fisici alla teoria ad onda pilota

[Einstein - S13 - p. 205]

[Vari fisici - S14 - pp. 205 - 206]

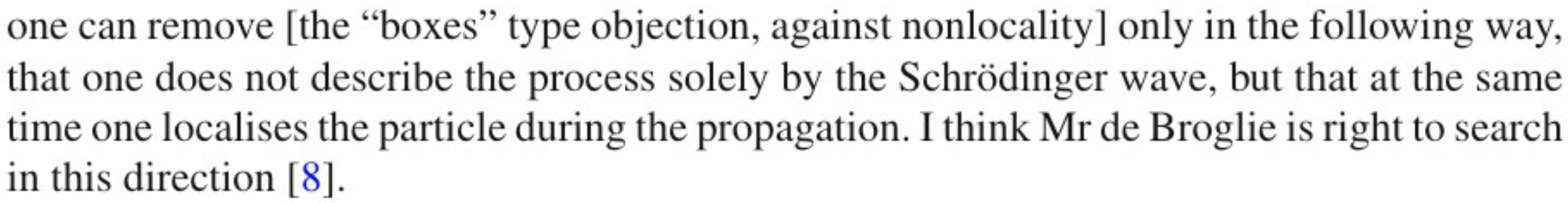
[Einstein - S15 - p. 206]

[Heisenberg - S16 - p. 207]

[Bell - S17 - p. 207]

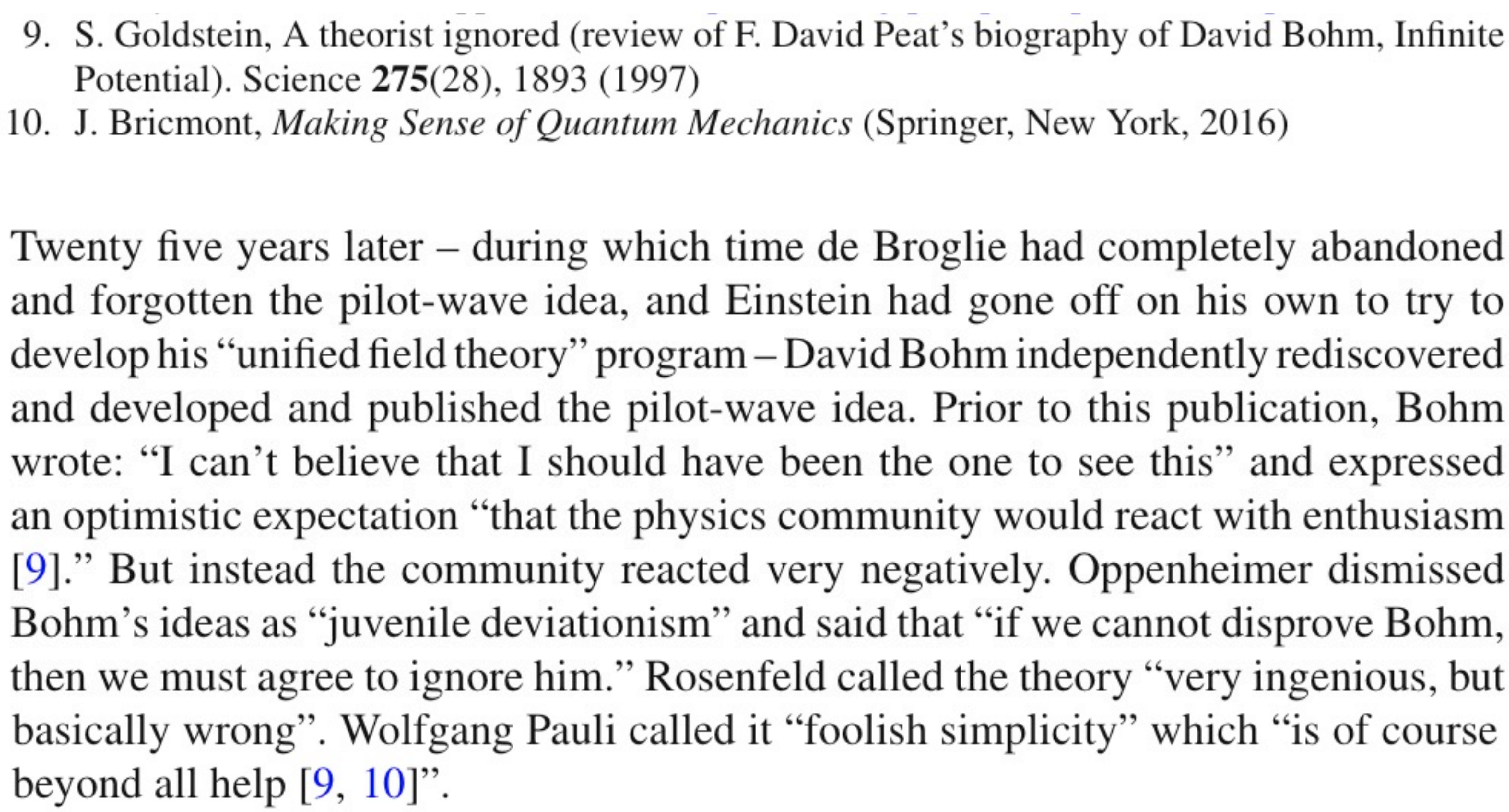
È possibile costruire un completamento a variabili nascoste della meccanica quantistica ordinaria che abbia le virtù di una teoria ad onda pilota senza il problema della non località?

8. Einstein's remarks from Solvay 1927, translated in Bacciogallupi and Valentini, Quantum theory at the crossroads, pp. 485–487, http://arxiv.org/pdf/quant-ph/0609184.pdf



one can remove [the "boxes" type objection, against nonlocality] only in the following way, that one does not describe the process solely by the Schrödinger wave, but that at the same time one localises the particle during the propagation. I think Mr de Broglie is right to search in this direction [8].

From "Foundations of Quantum Mechanics" by Travis Norsen, Springer 2017.



9. S. Goldstein, A theorist ignored (review of F. David Peat's biography of David Bohm, Infinite Potential). Science **275**(28), 1893 (1997)
10. J. Bricmont, *Making Sense of Quantum Mechanics* (Springer, New York, 2016)

Twenty five years later – during which time de Broglie had completely abandoned and forgotten the pilot-wave idea, and Einstein had gone off on his own to try to develop his "unified field theory" program – David Bohm independently rediscovered and developed and published the pilot-wave idea. Prior to this publication, Bohm wrote: "I can't believe that I should have been the one to see this" and expressed an optimistic expectation "that the physics community would react with enthusiasm [9]." But instead the community reacted very negatively. Oppenheimer dismissed Bohm's ideas as "juvenile deviationism" and said that "if we cannot disprove Bohm, then we must agree to ignore him." Rosenfeld called the theory "very ingenious, but basically wrong". Wolfgang Pauli called it "foolish simplicity" which "is of course beyond all help [9, 10]".

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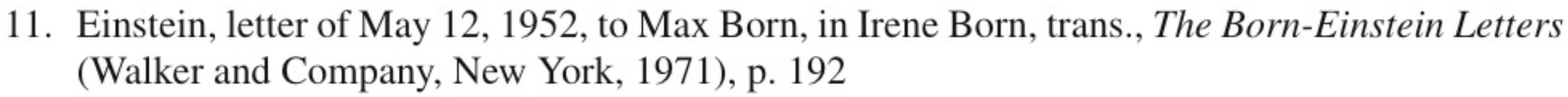
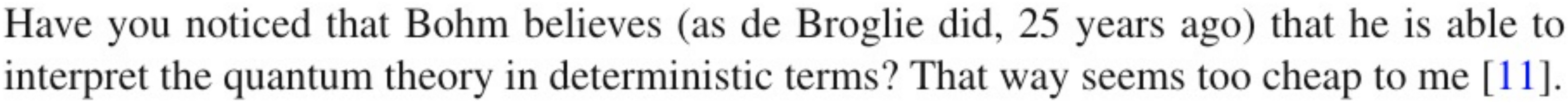


11. Einstein, letter of May 12, 1952, to Max Born, in Irene Born, trans., *The Born-Einstein Letters* (Walker and Company, New York, 1971), p. 192



Have you noticed that Bohm believes (as de Broglie did, 25 years ago) that he is able to interpret the quantum theory in deterministic terms? That way seems too cheap to me [11].

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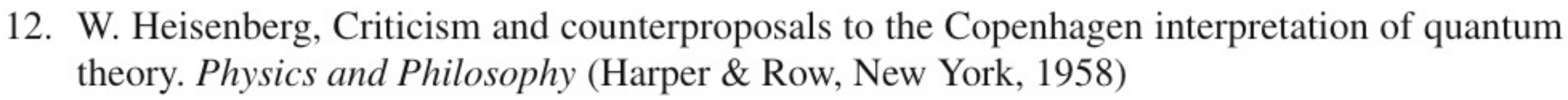
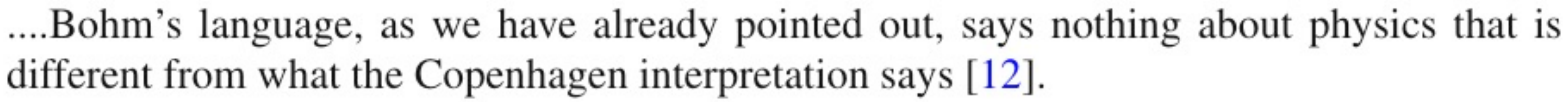


12. W. Heisenberg, Criticism and counterproposals to the Copenhagen interpretation of quantum theory. *Physics and Philosophy* (Harper & Row, New York, 1958)



....Bohm's language, as we have already pointed out, says nothing about physics that is different from what the Copenhagen interpretation says [12].

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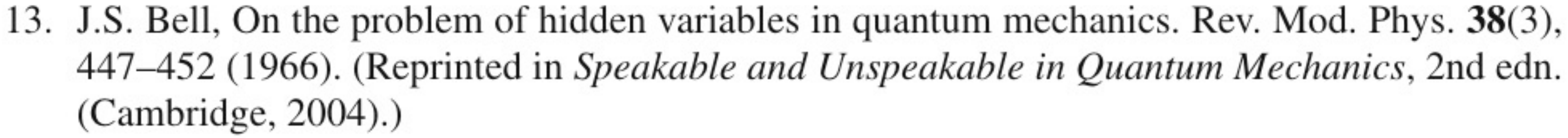
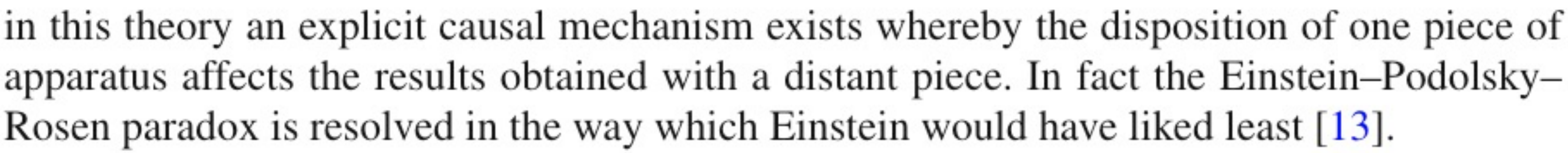


13. J.S. Bell, On the problem of hidden variables in quantum mechanics. Rev. Mod. Phys. **38**(3), 447–452 (1966). (Reprinted in *Speakable and Unspeakable in Quantum Mechanics*, 2nd edn. (Cambridge, 2004).)



in this theory an explicit causal mechanism exists whereby the disposition of one piece of apparatus affects the results obtained with a distant piece. In fact the Einstein–Podolsky–Rosen paradox is resolved in the way which Einstein would have liked least [13].

From “Foundations of Quantum Mechanics” by Travis Norsen, Springer 2017.

Quantum Analogs with Walking Droplets

Scientific question

To what extent can a **classical** *wave-driven* system reproduce phenomena usually thought to be peculiar to the **quantum** realm?

Try to respond with **experiments**

(and experiment-inspired models).

The Faraday instability

Faraday threshold γ_F

Forcing acceleration $\Gamma(t) = \gamma \cos(2\pi f_0 t)$

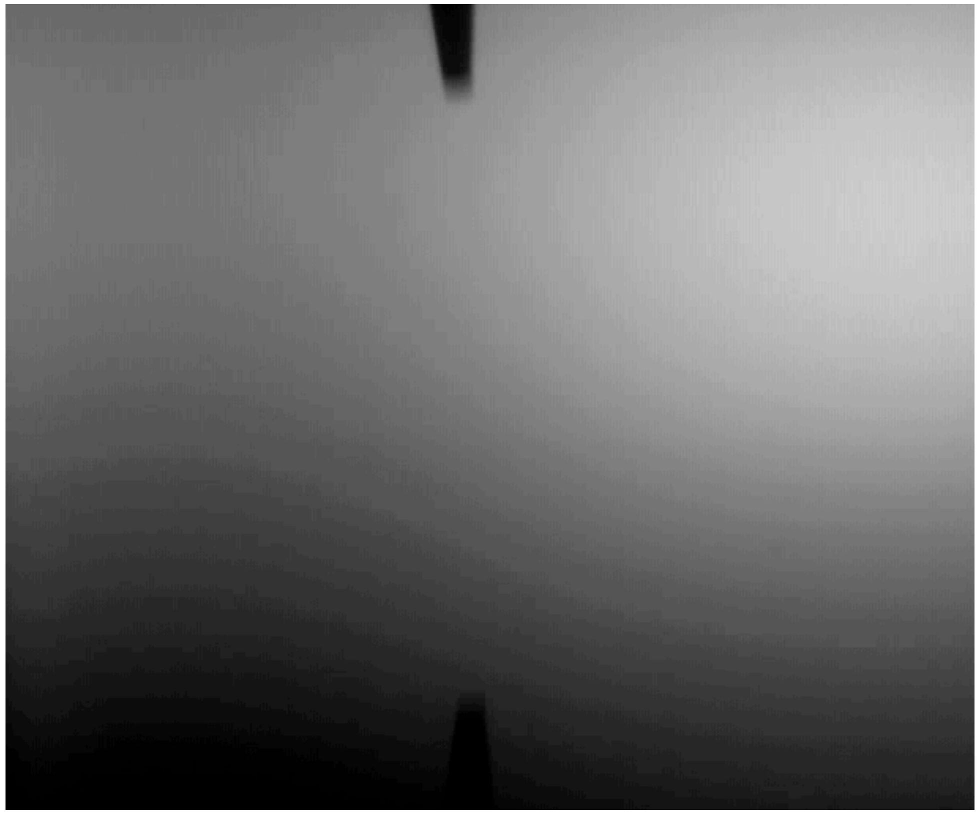
$f_0 \sim 100 \text{ Hz}$

Waves are subharmonic and standing $f = f_0/2$

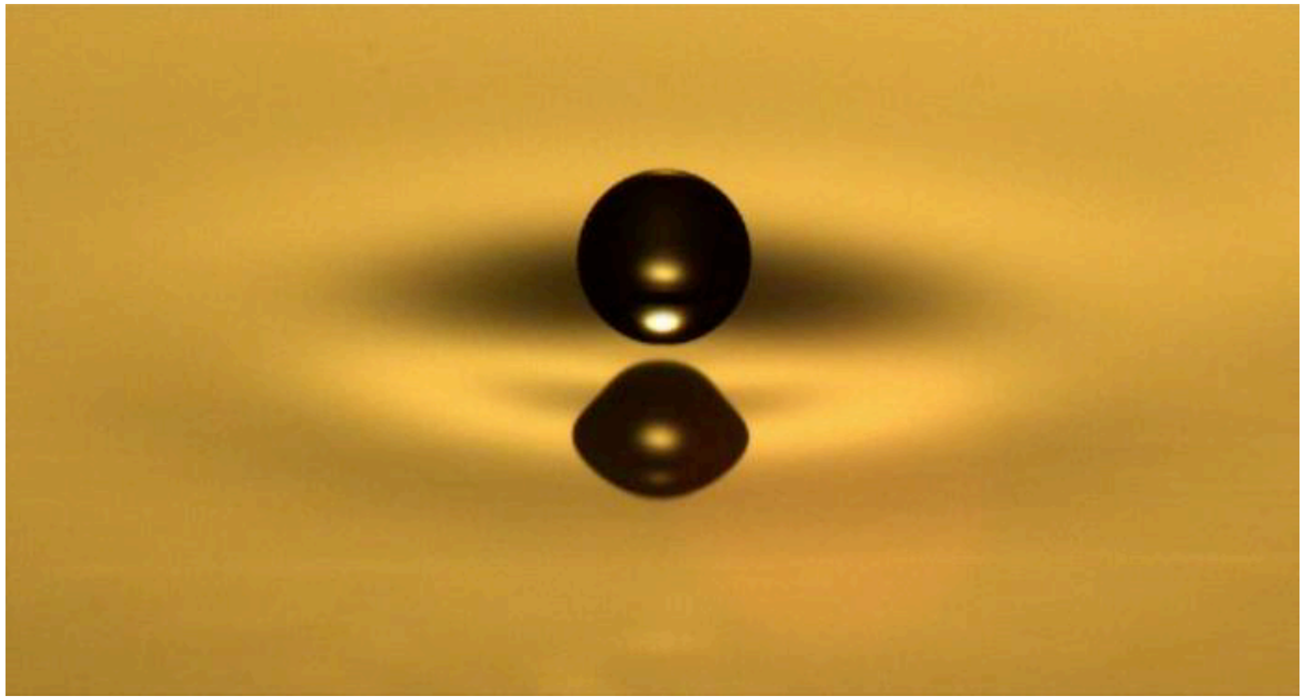
Bouncing drops

$$\Gamma(t) = \gamma \cos(2\pi f_0 t)$$

Below the Faraday threshold $\gamma < \gamma_F$



Slow motion video from Couder & Fort's group in Paris



Slow motion video from Bush's group at MIT

silicone oil with viscosity 20 cSt $f_0 = 80 \text{ Hz}$ $\gamma \sim 3 \text{ g}$

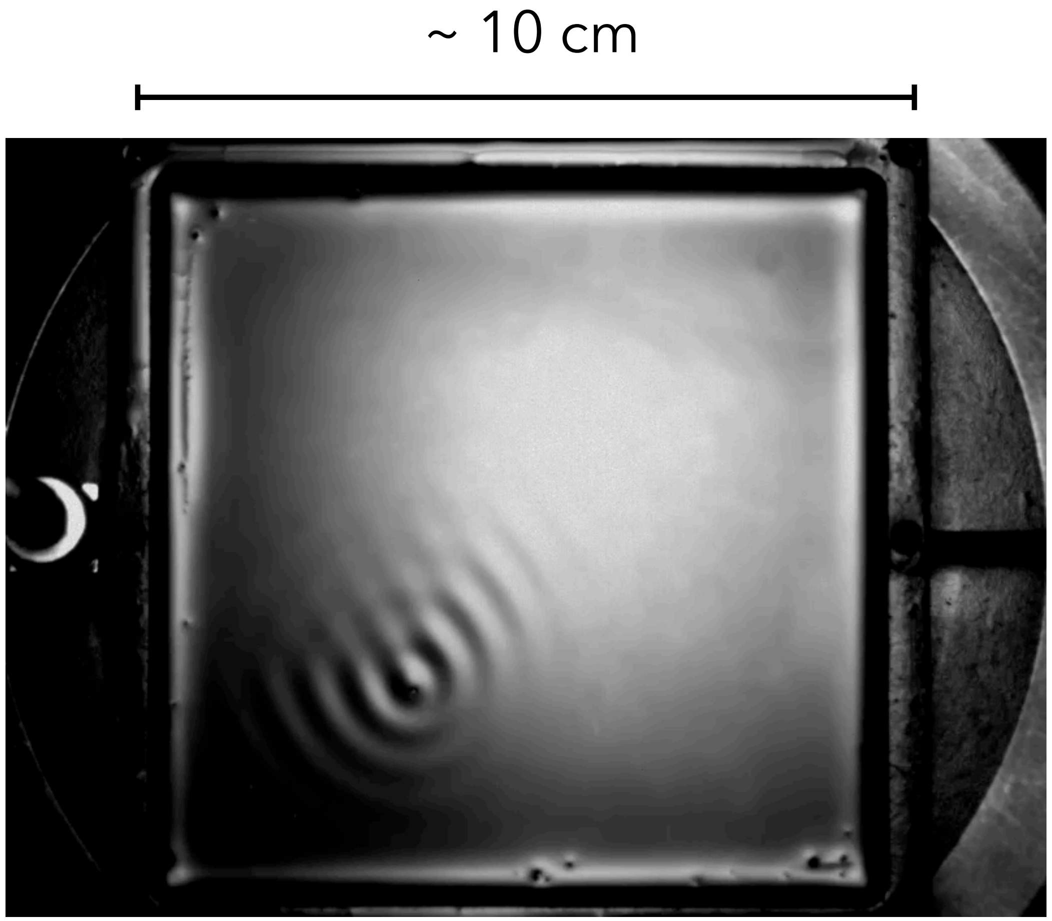
- coalescence avoided provided impact time is less than time required for air layer between drop and bath to drain to ~100 nm

J. Walker (1978), Couder *et al. Phys. Rev. Lett.* (2005).

Walking droplets

- Yves Couder, Emmanuel Fort and coworkers in Paris discovered that if the forcing amplitude is increased further, the droplet can begin to translate steadily across the surface propelled by its self-generated wave field.
- spatially extended **walkers** composed of **droplet** and **monochromatic wave**

Slow motion video from Bush's group at MIT



Real time, top view video from Couder's group in Paris

Couder, Y. *et al.* *Nature* (2005) / Bush, J. W. M. *Ann. Rev. Fluid Mech.* (2015).

Wave field with increasing forcing amplitude: Memory

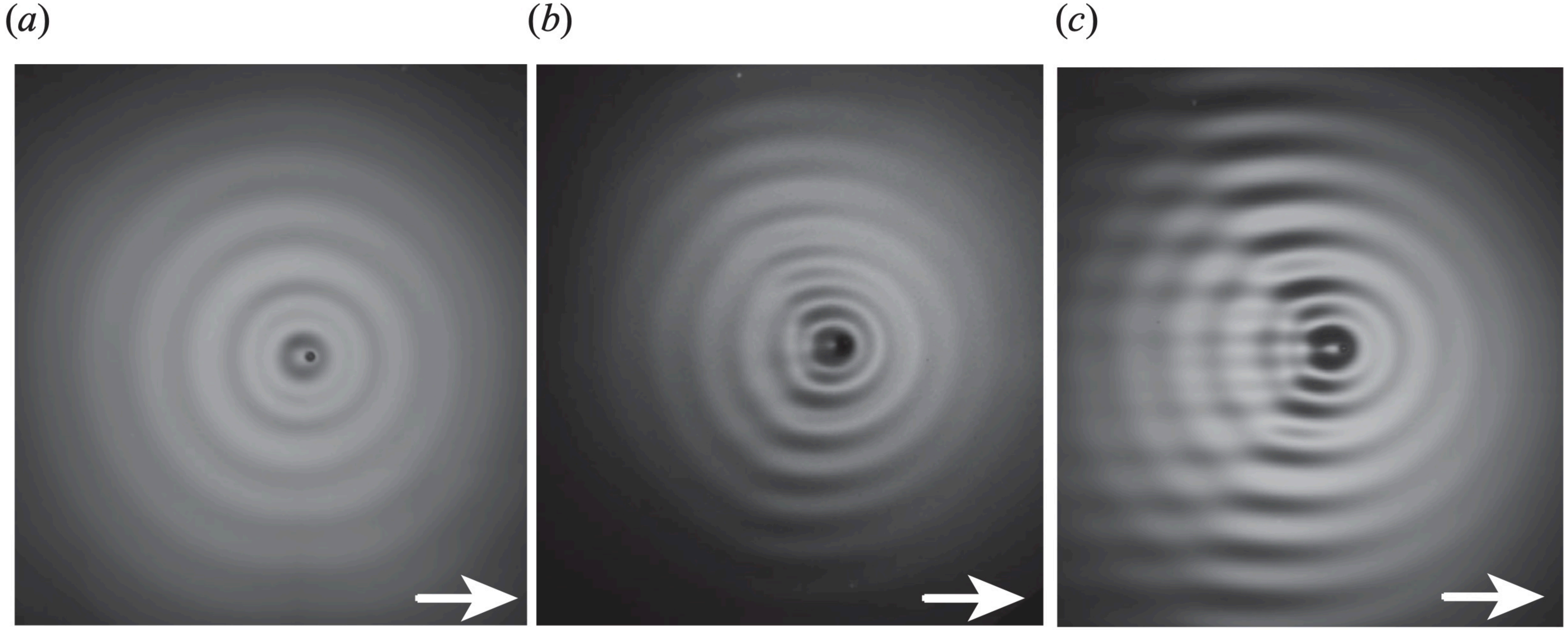
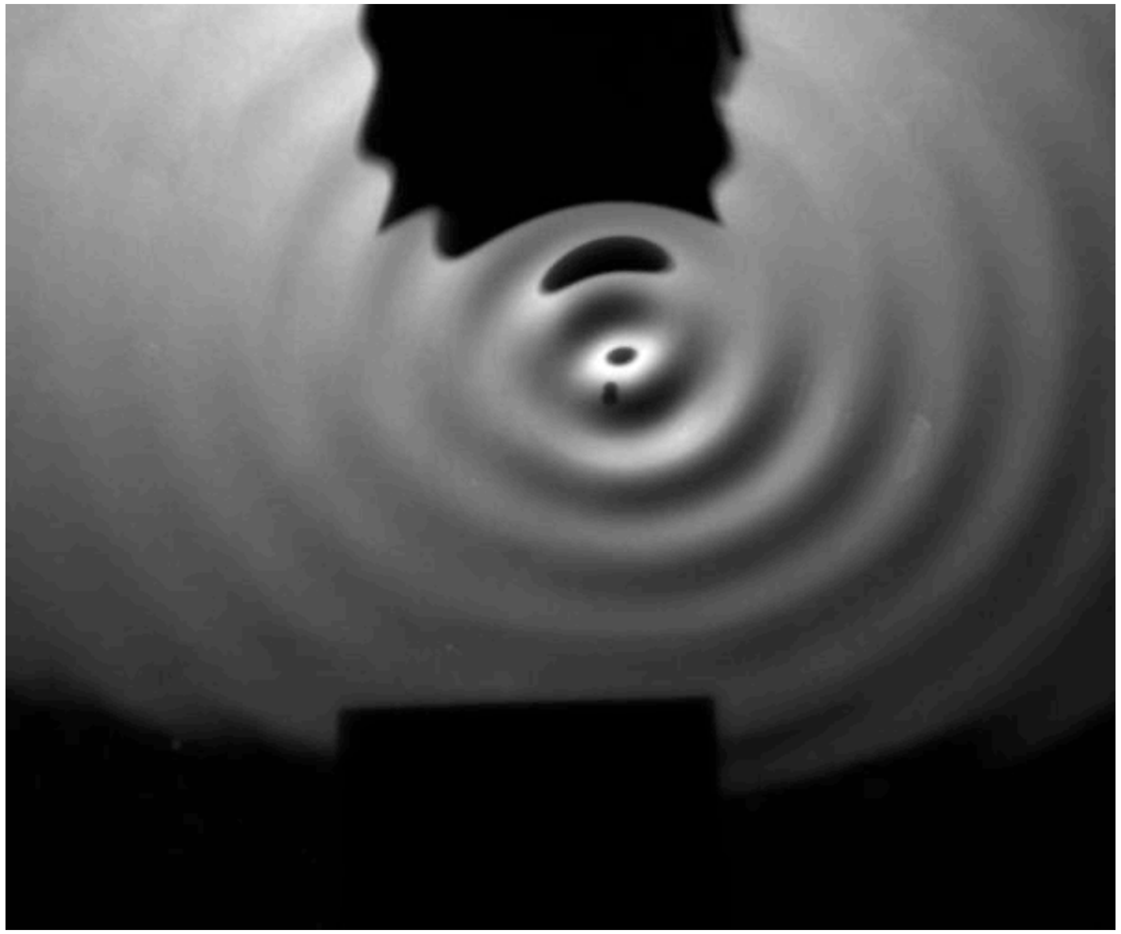


FIGURE 2. Photographs of the wave field associated with the same droplet of diameter $D = 0.76$ mm for three different forcing accelerations in the walking regime (points a, b and c of the phase diagram, see figure 1). The drop propagates from left to right. (*a*) Wave field observed at point a, where $\Gamma = (\gamma_F - \gamma)/\gamma_F = 0.17$, i.e. $\Gamma^{-1} = 5.9$. (*b*) Wave field at b, where $\Gamma = 0.07$, i.e. $\Gamma^{-1} = 14.3$. (*c*) Wave field at c, where $\Gamma = 0.018$, i.e. $\Gamma^{-1} = 55.5$. The value of Γ^{-1} gives an order of magnitude of the number of bounces which contribute to the wave field interference pattern.

$$Me = \frac{\tau}{T_F} = \frac{T_d}{T_F(1 - \gamma/\gamma_F)}$$

A. Eddi *et al. J. Fluid Mech.* 674:433-463 (2011).

Orbital quantization and eigenstates



Angular rotation speed as an effective magnetic field

Electromagnetism $\vec{B} = \vec{\nabla} \wedge \vec{A}$ $\vec{F}_B = q(\vec{v} \wedge \vec{B})$

Fluid mechanics $2\vec{\Omega} = \vec{\nabla} \wedge \vec{U}$ $\vec{F}_\Omega = -m(\vec{v} \wedge 2\vec{\Omega})$

Berry et al. 1980.

Path-memory induced quantization of classical orbits

Emmanuel Fort[a,1], Antonin Eddi[b], Arezki Boudaoud[c], Julien Moukhtar[b], and Yves Couder[b]

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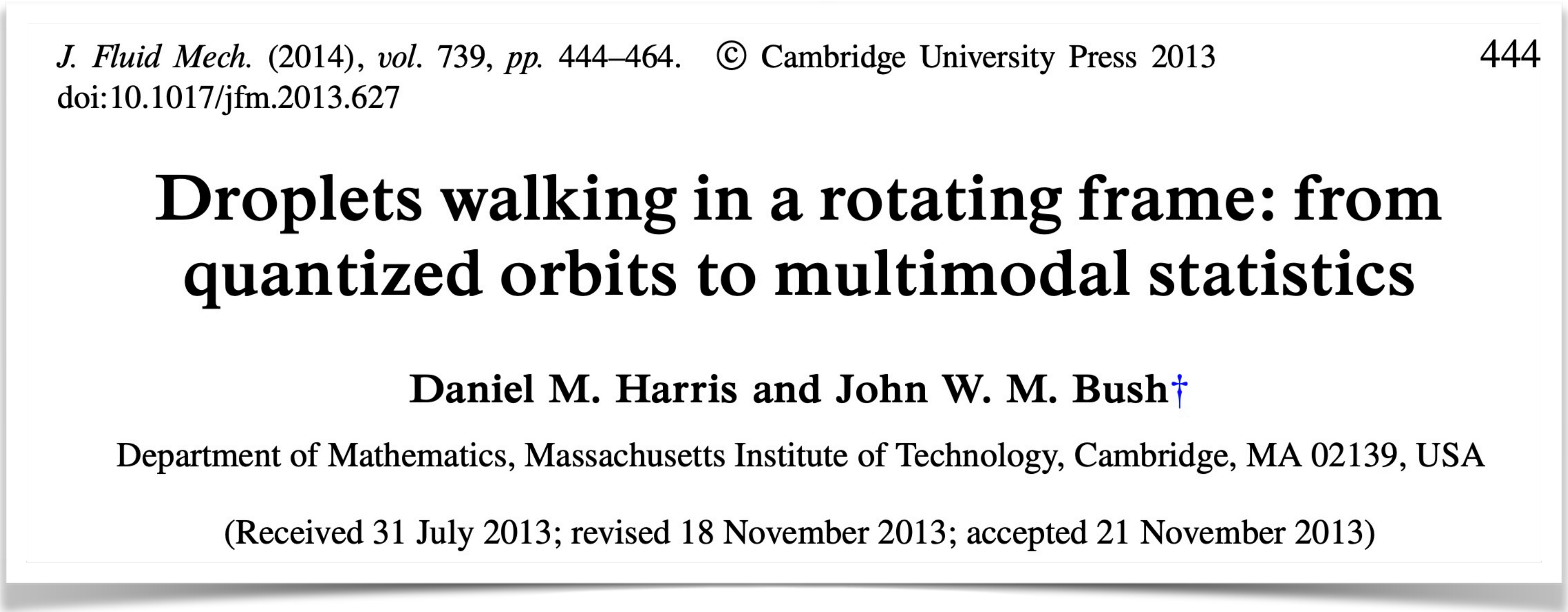

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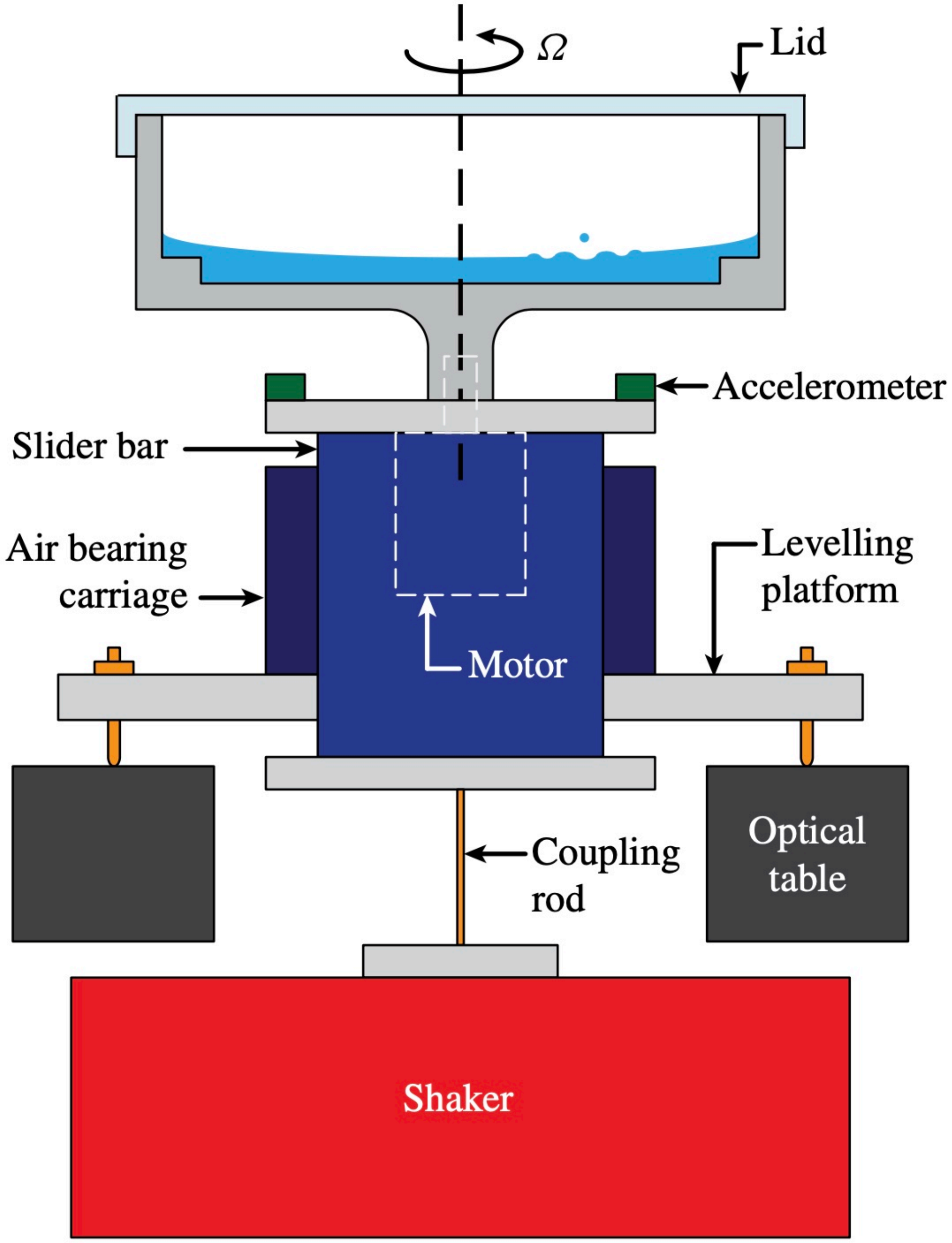

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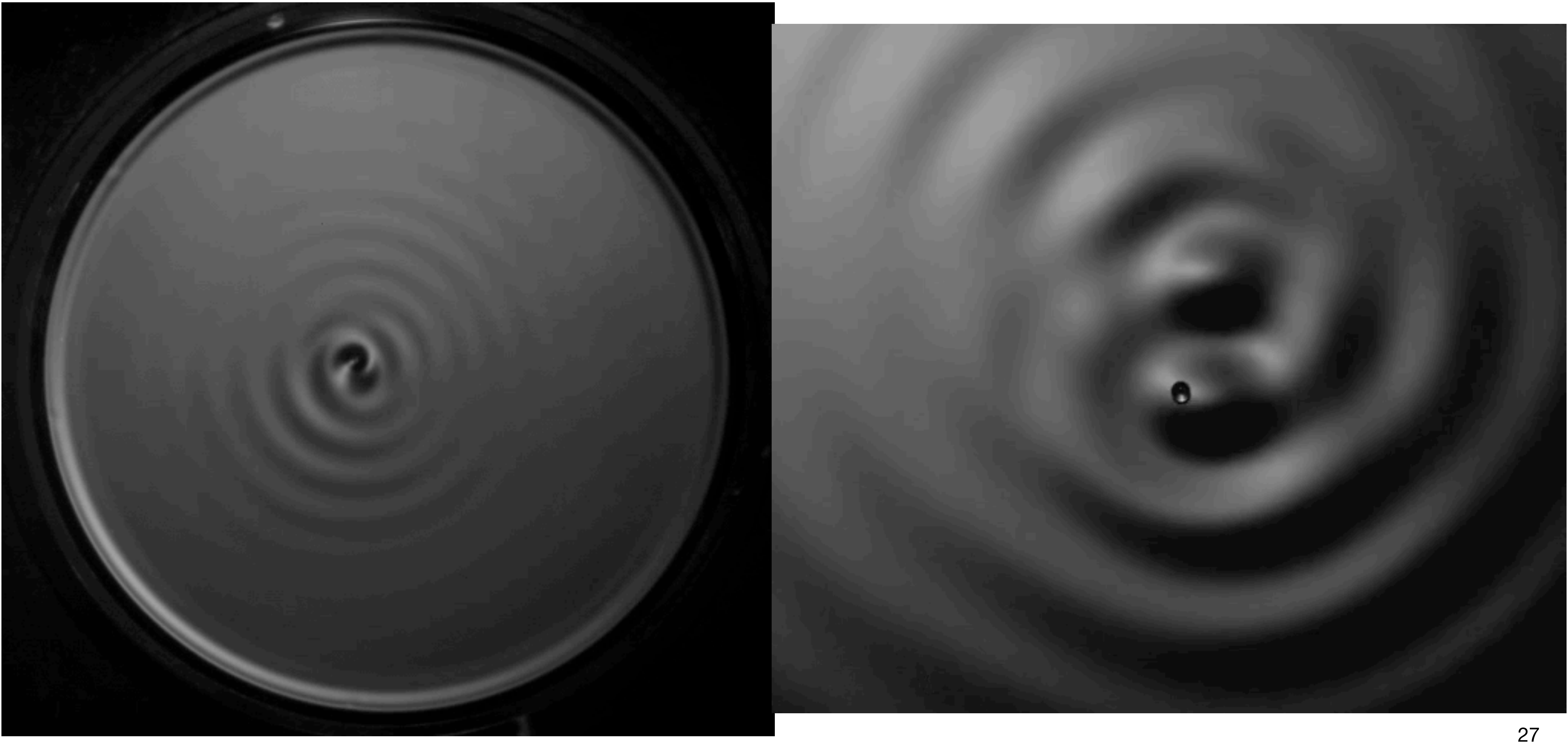

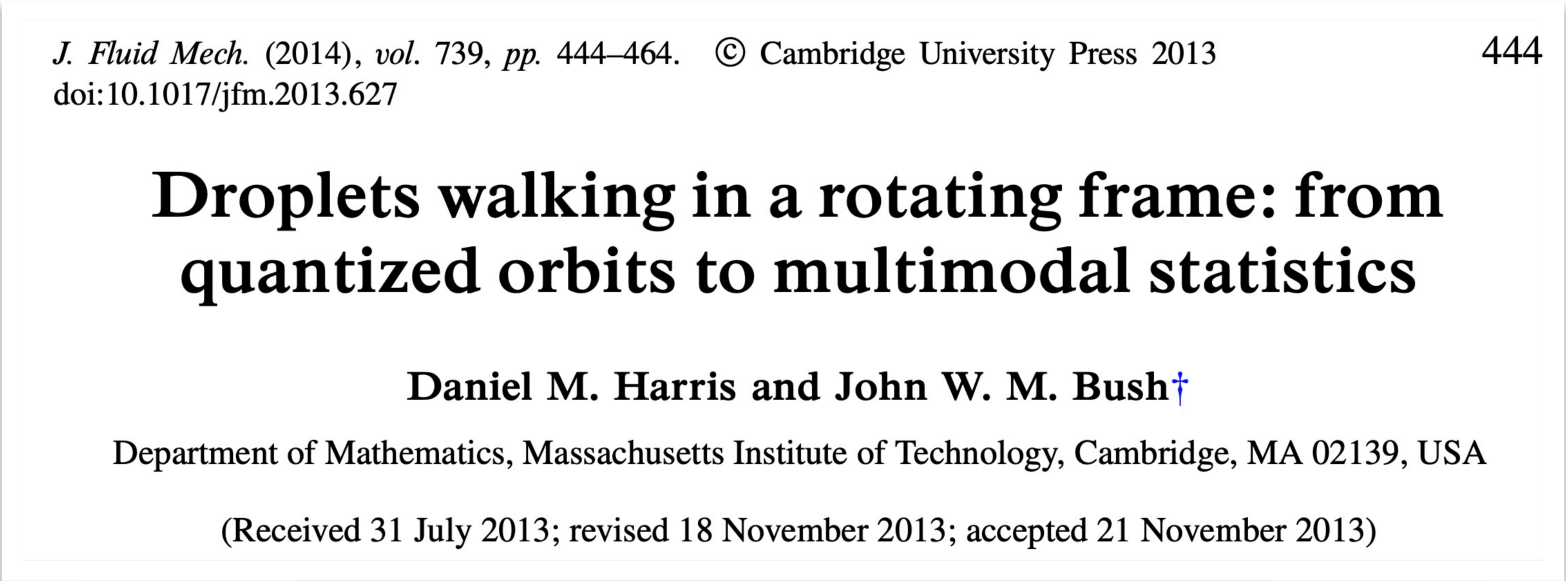

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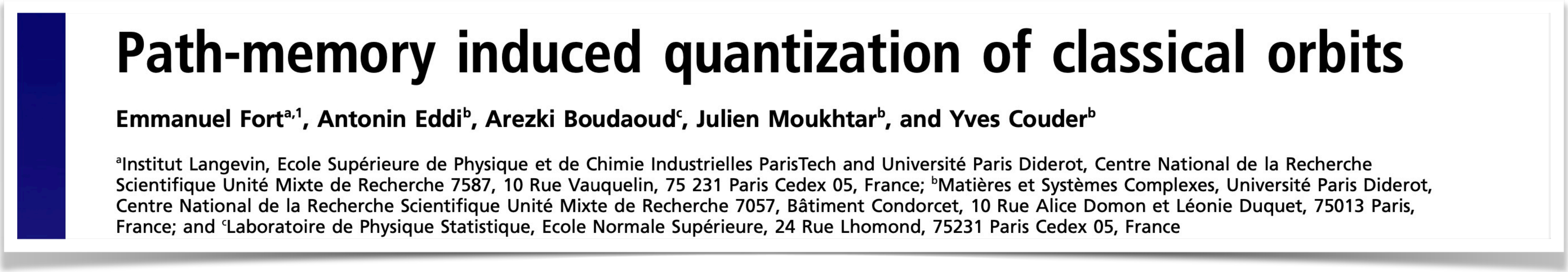

Path-memory induced quantization of classical orbits

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In a magnetic field the orbit has a radius

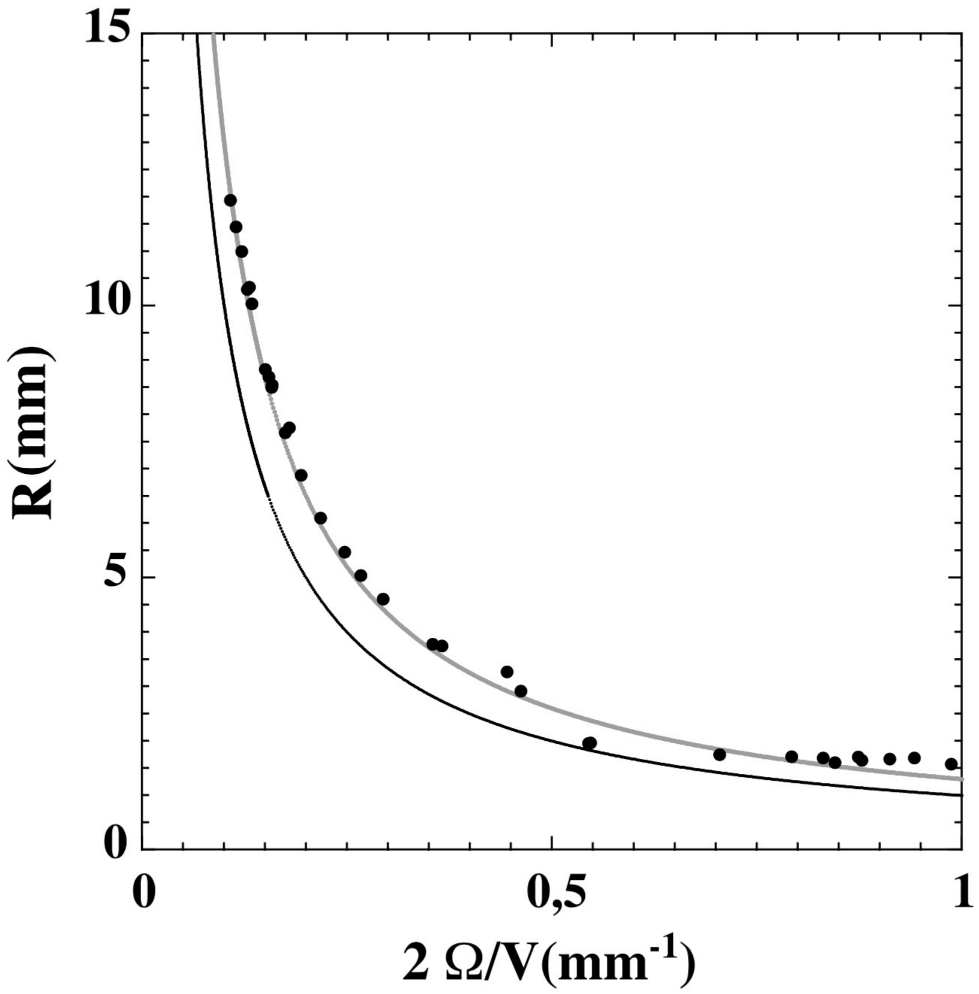
$$\rho_L = mv/qB \qquad [1]$$

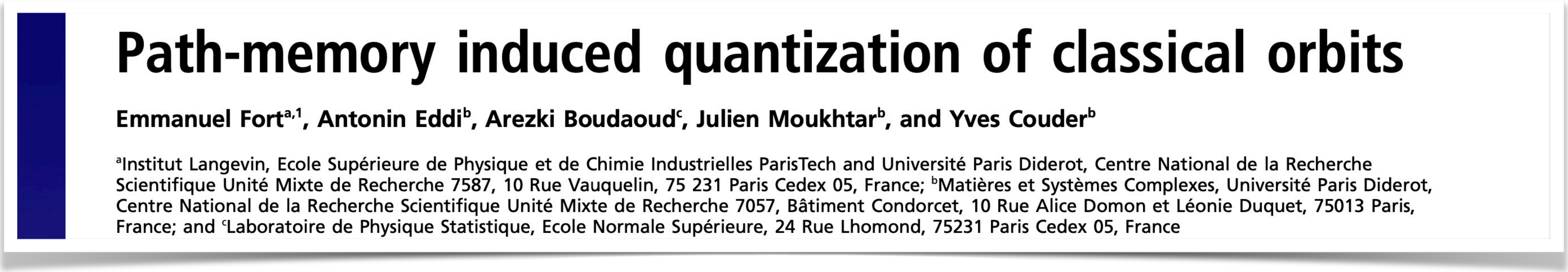
and a Larmor period $\tau_L = m/qB$. On a rotating surface a mobile particle with velocity V moves on a circle of radius

$$R_C = V/2\Omega \qquad [2]$$

For short-term memory, the walker moves on an orbit of decreasing radius R_C (Fig. 2*A*) as would be expected from Eq. **2**. The measured values are, however, larger than expected and we find an excellent fit with

$$R_C^{\mathrm{exp}} = a(V_W/2\Omega), \qquad [3]$$





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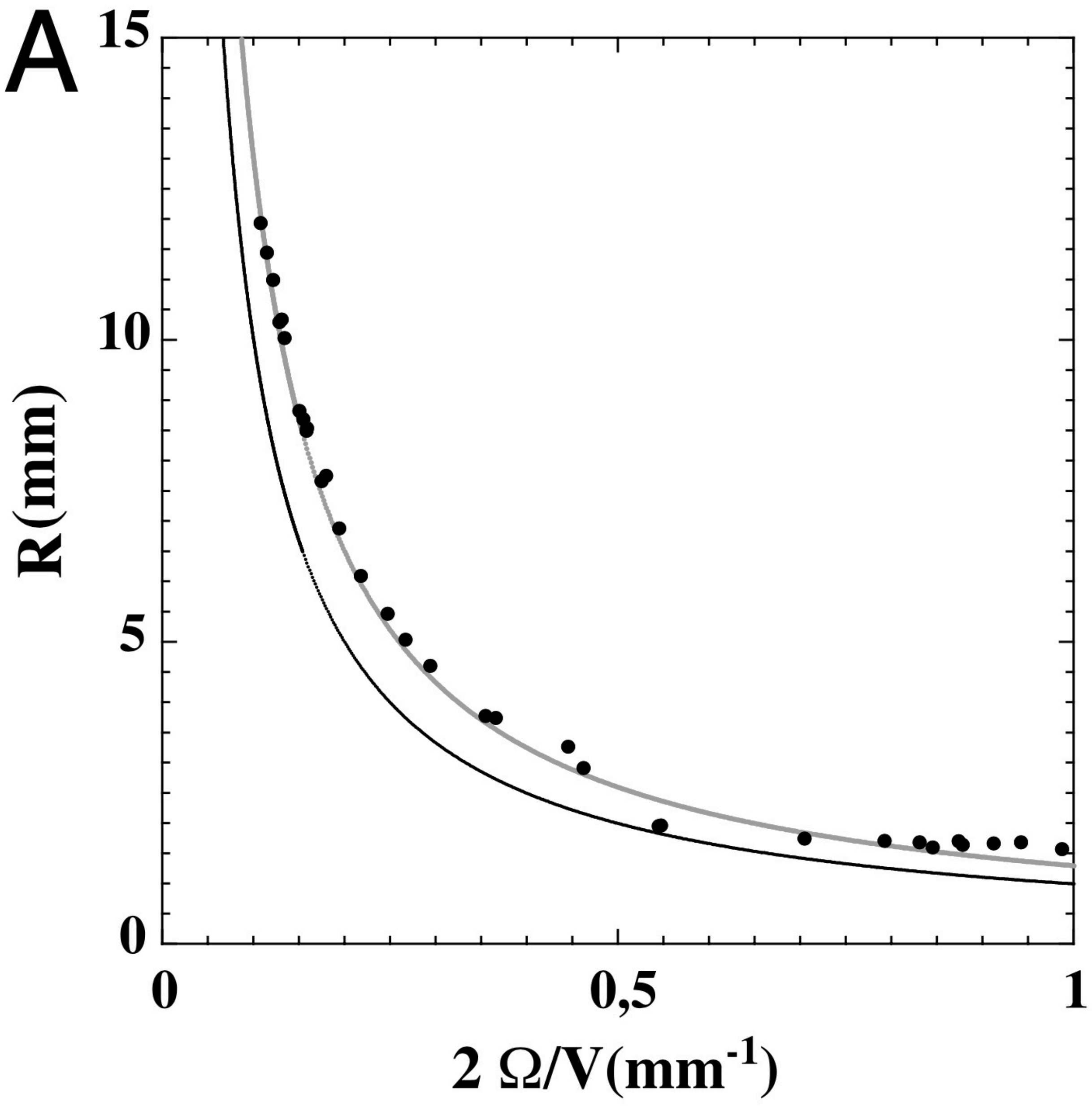
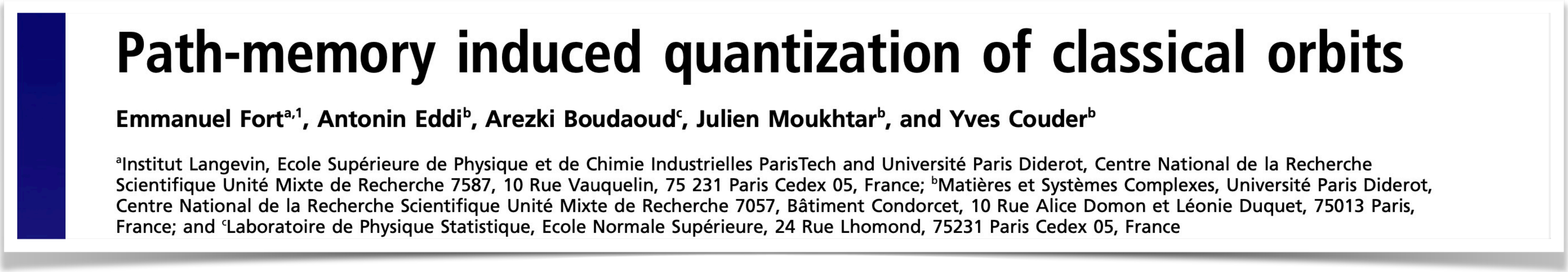
low memory

high memory





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Let us now look for an analogy between the discretization in our results and that of quantum systems. In quantum mechanics, the motion of a charge in a magnetic field is quantified in Landau levels (15). The associated Larmor radius only takes discrete values ρ_n given by

$$\rho_n = \sqrt{1/\pi}\sqrt{\left(n+\frac{1}{2}\right)\frac{h}{qB}} \qquad [4]$$

with $n = 0,1,2,\ldots$. This equation can alternatively be expressed as a function of the de Broglie wavelength $\lambda_{\mathrm{dB}} = h/mV$. The radii then satisfy

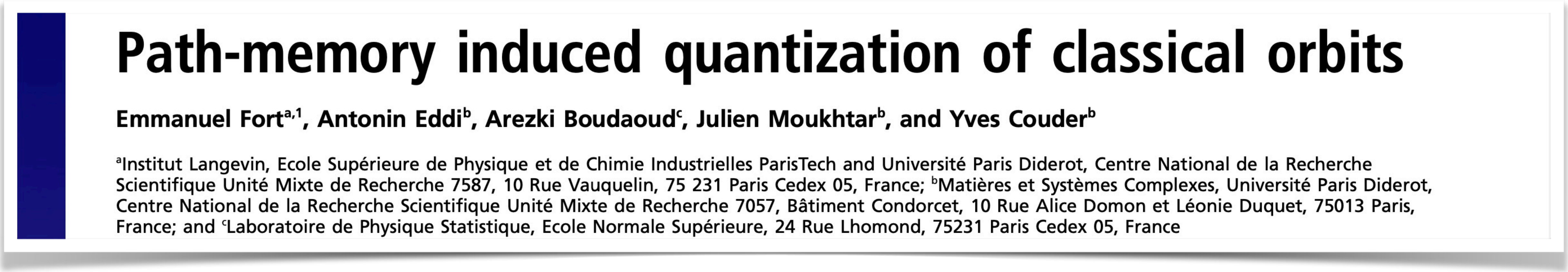
$$\frac{\rho_n}{\lambda_{\mathrm{dB}}} = \sqrt{1/\pi}\sqrt{\left(n+\frac{1}{2}\right)\frac{m}{qB}\frac{V}{\lambda_{\mathrm{dB}}}}. \qquad [5]$$

The Landau orbits coincide with the classical ones when Eqs. **1** and **5** are satisfied simultaneously. Eliminating mV/qB gives

$\rho_L = mv/qB$

$$\rho_n = \frac{1}{\pi}\left(n+\frac{1}{2}\right)\lambda_{\mathrm{dB}}, \qquad [6]$$

which corresponds to the Bohr–Sommerfeld quantization of the orbits perimeter.



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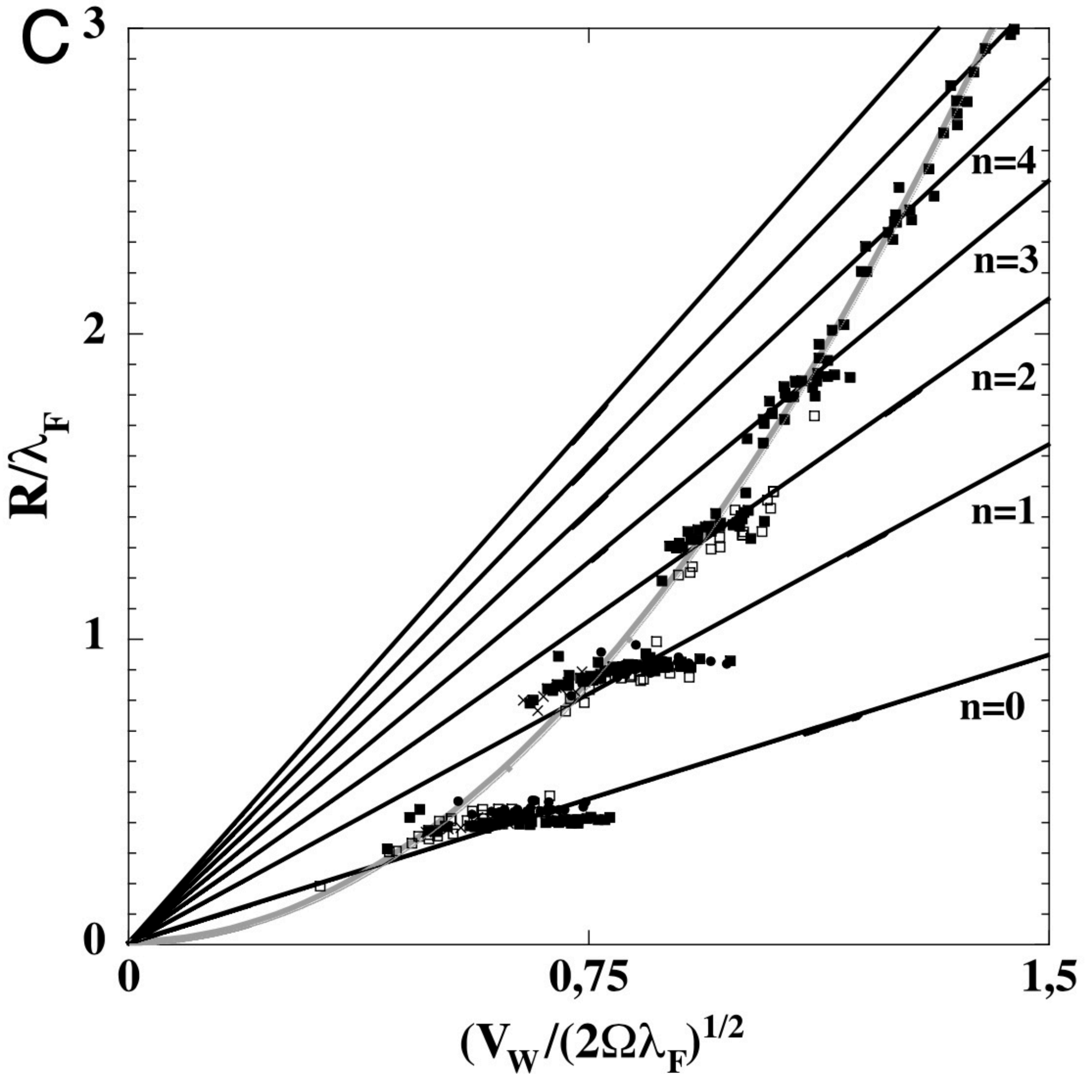
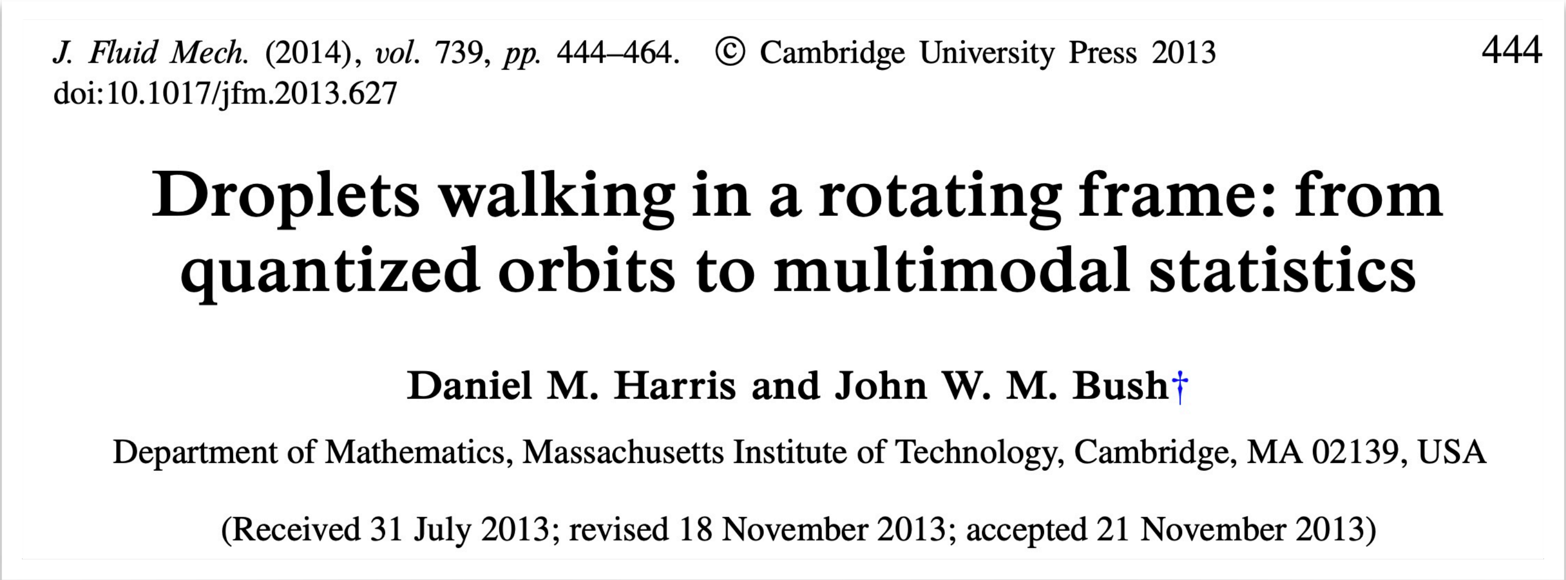
We can now ask ourselves if our experimental results have some analogy with this quantization by first using the previously discussed correspondence where $1/2\Omega$ is the analogue of the Larmor period $\tau_L = m/qB$. A bolder step is to assume that in our classical system the Faraday wavelength could play a role comparable to the de Broglie wavelength in the quantum situation. We thus try fitting our data with a dependence of the type given by

$$\frac{R_n}{\lambda_F} = b\sqrt{\left(n+\frac{1}{2}\right)\frac{1}{2\Omega}\frac{V_W}{\lambda_F}}. \qquad [7]$$

$$\frac{\rho_n}{\lambda_{\mathrm{dB}}} = \sqrt{1/\pi}\sqrt{\left(n+\frac{1}{2}\right)\frac{m}{qB}\frac{V}{\lambda_{\mathrm{dB}}}}. \qquad [5]$$





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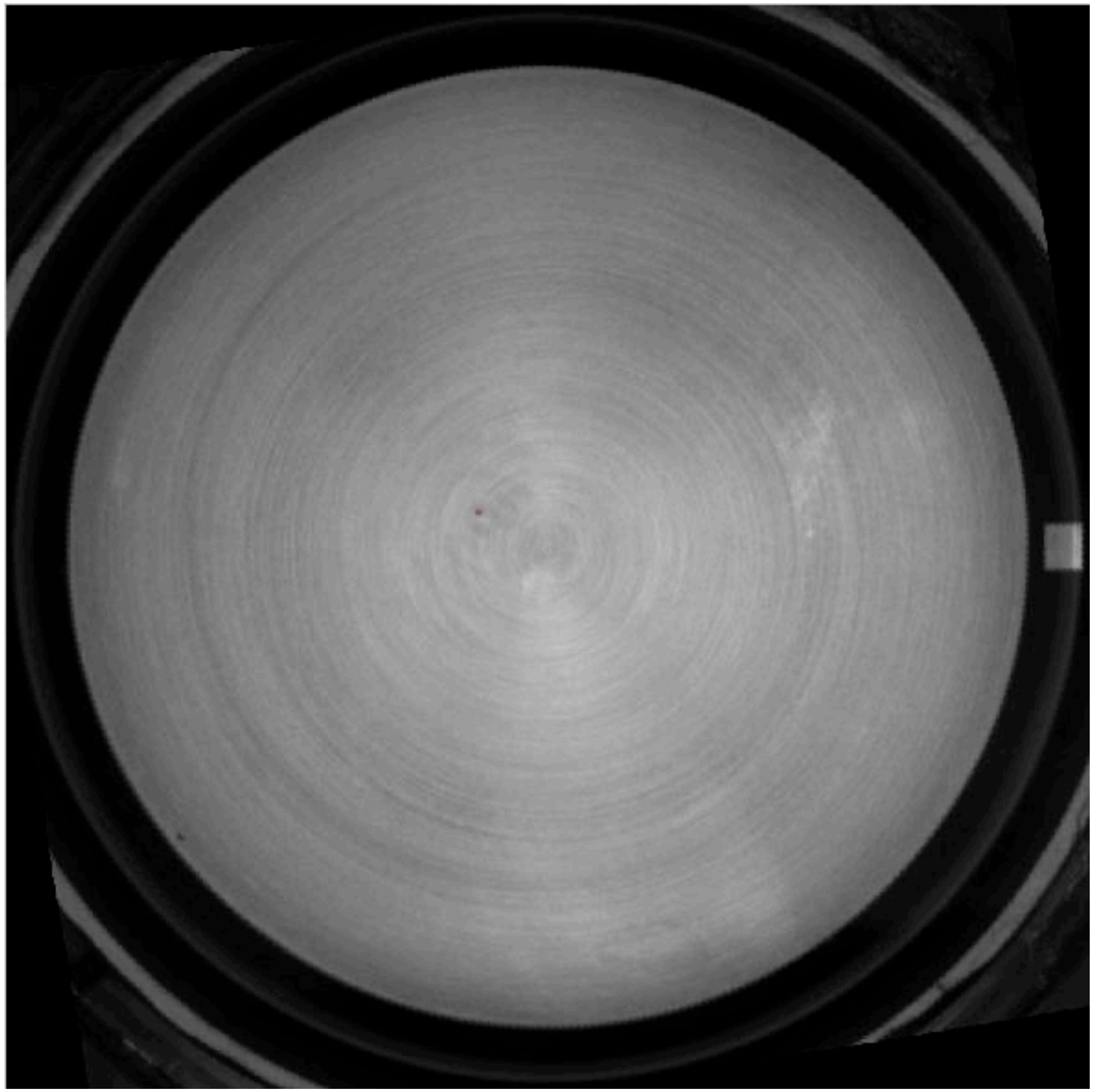
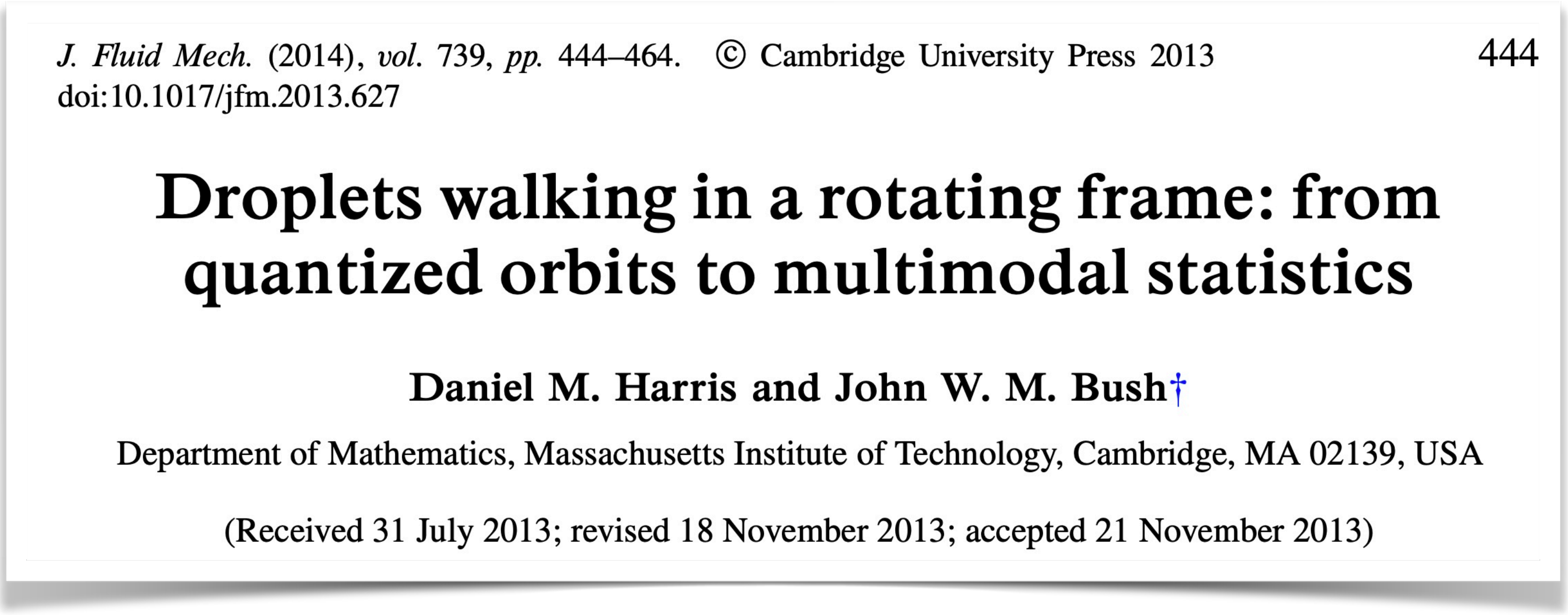
wobbling





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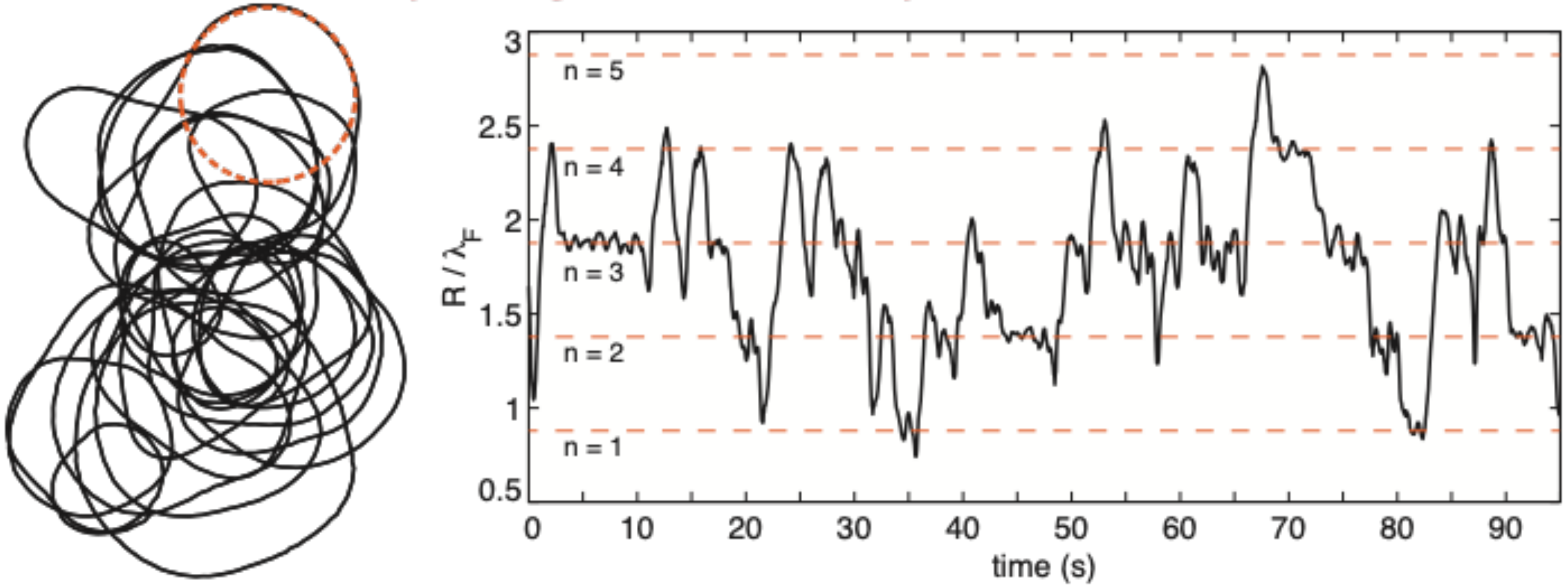
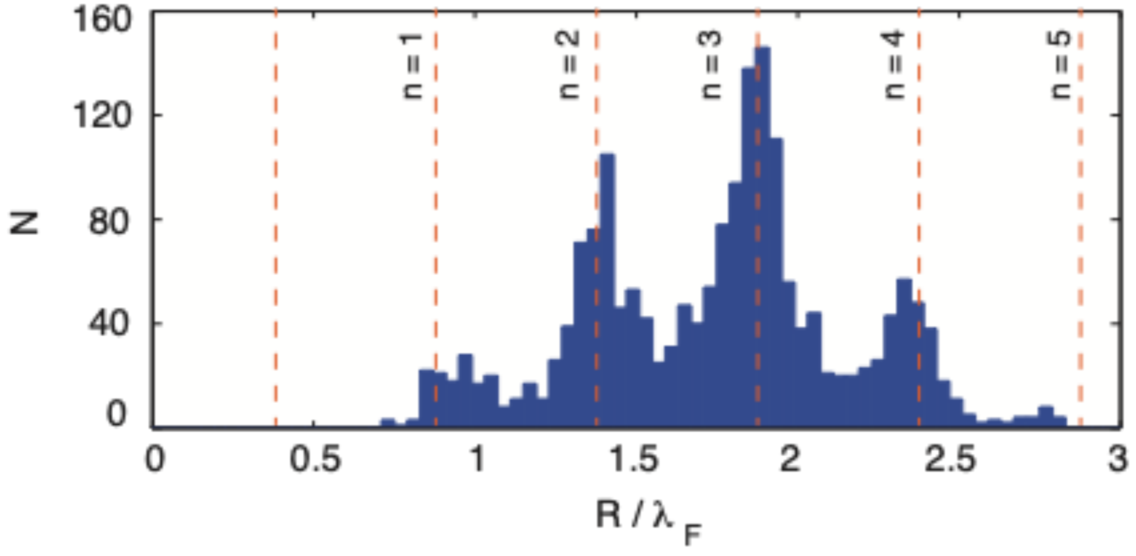
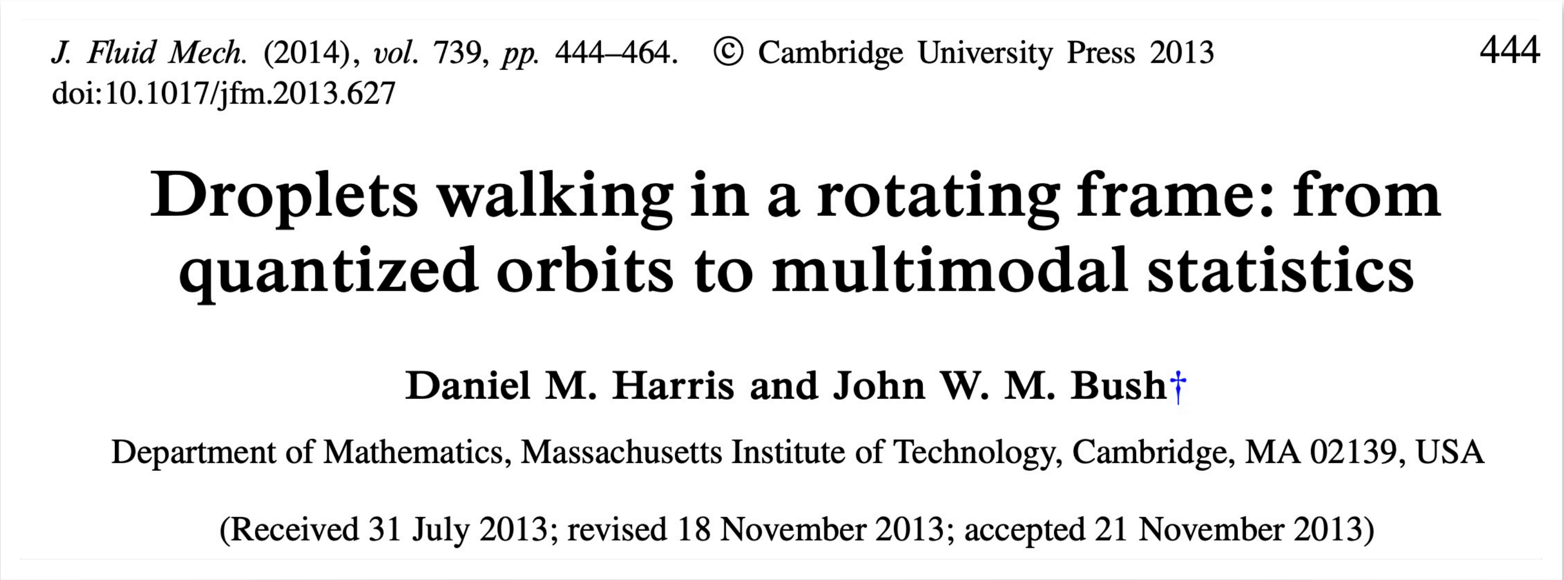
Very high memory behavior





The walker switches between arcs with radius corresponding to the unstable quantized orbits.



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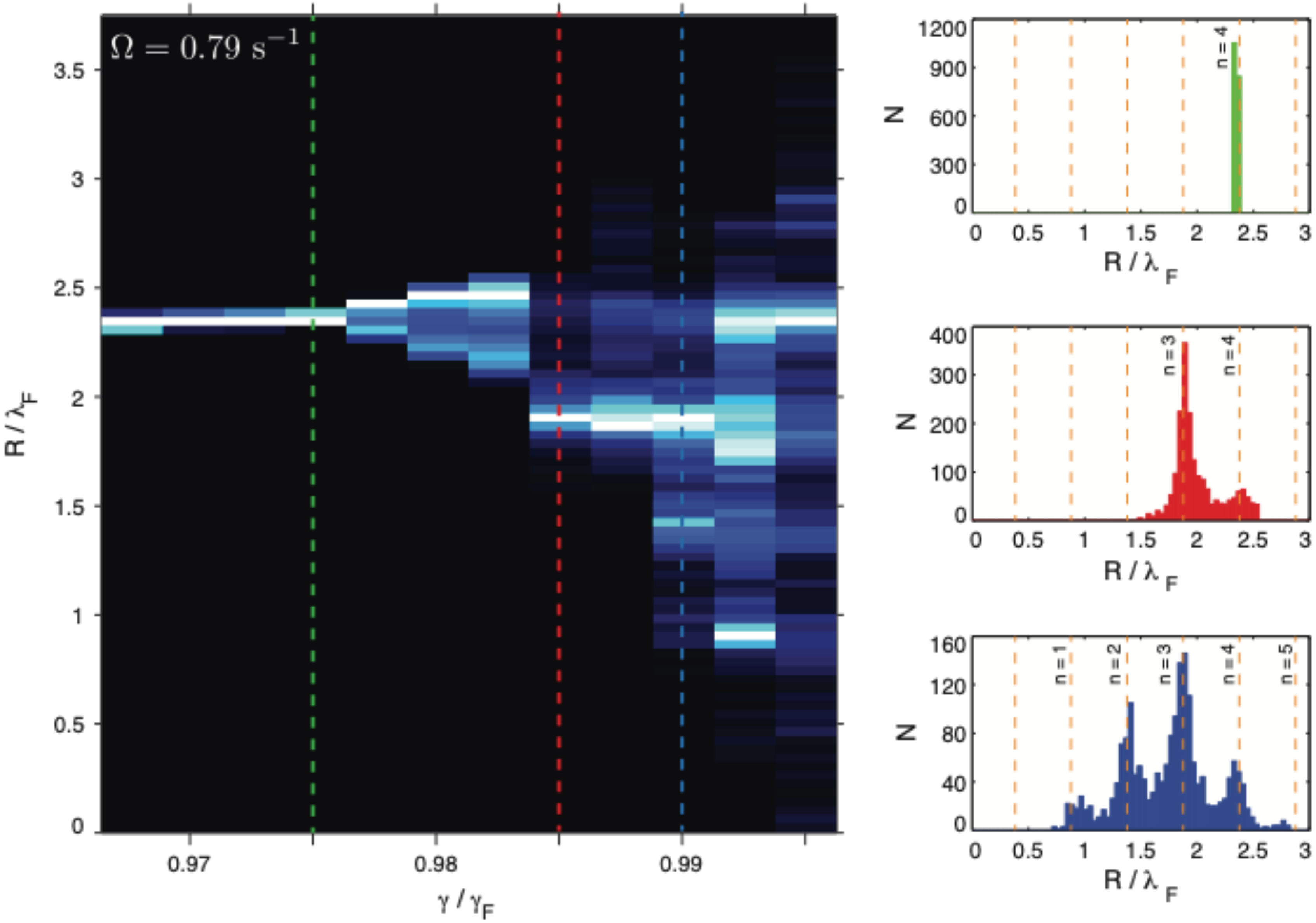
(Received 31 July 2013; revised 18 November 2013; accepted 21 November 2013)

Evolution of statistical behavior



Quantum 1D harmonic oscillator

(Source: Wikipedia)

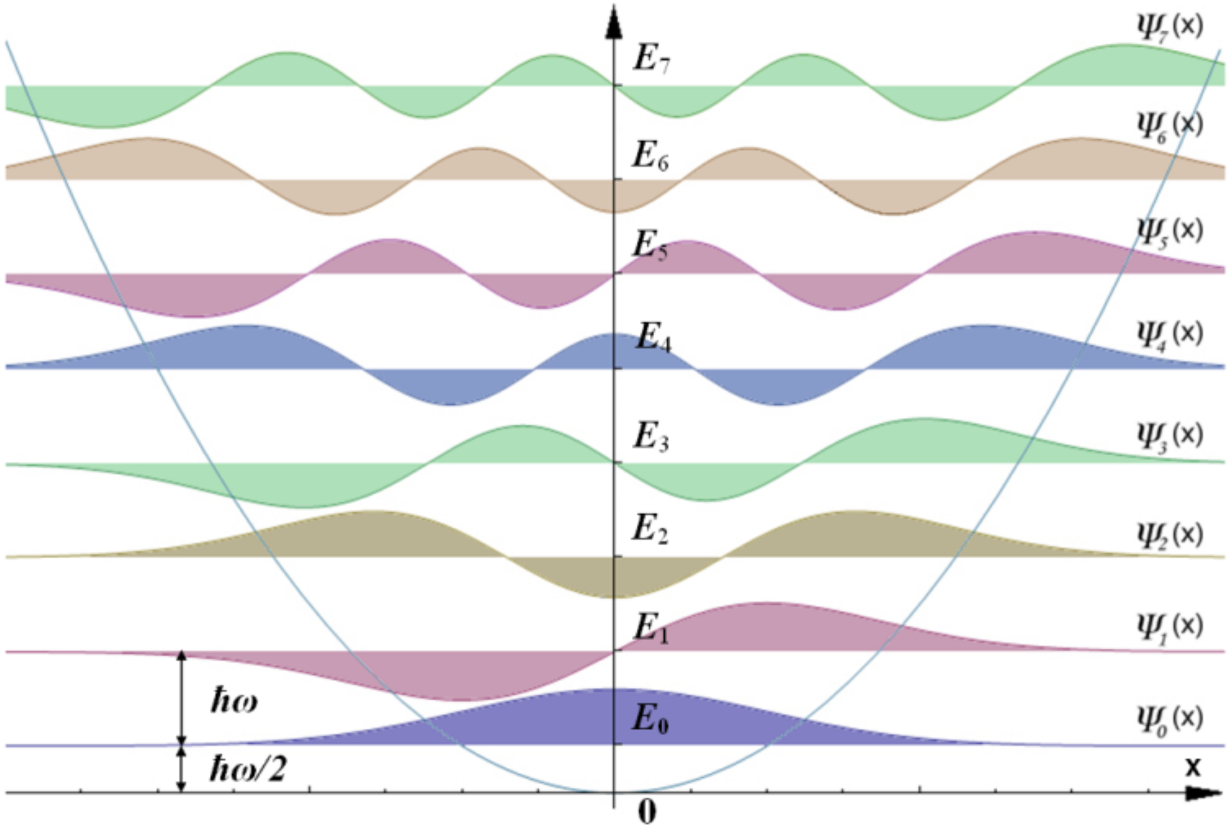
$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

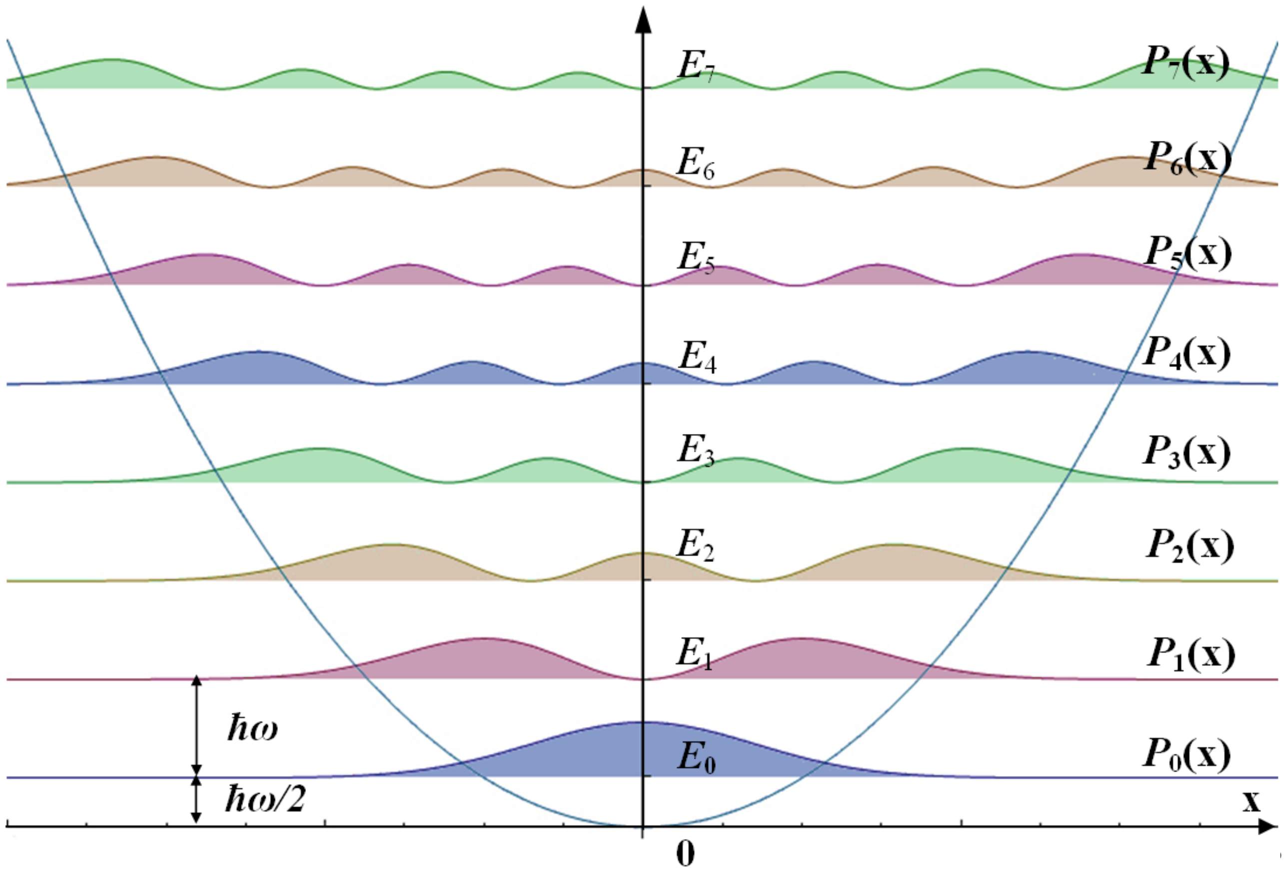
$$\hat{H}\,|\psi\rangle = E\,|\psi\rangle$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n\,n!}} \cdot \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right), \qquad n = 0, 1, 2, \ldots.$$

$$H_n(z) = (-1)^n\, e^{z^2} \frac{d^n}{dz^n}\left(e^{-z^2}\right)$$

$$E_n = \hbar\omega\big(n + \tfrac{1}{2}\big) = (2n+1)\frac{\hbar}{2}\omega$$





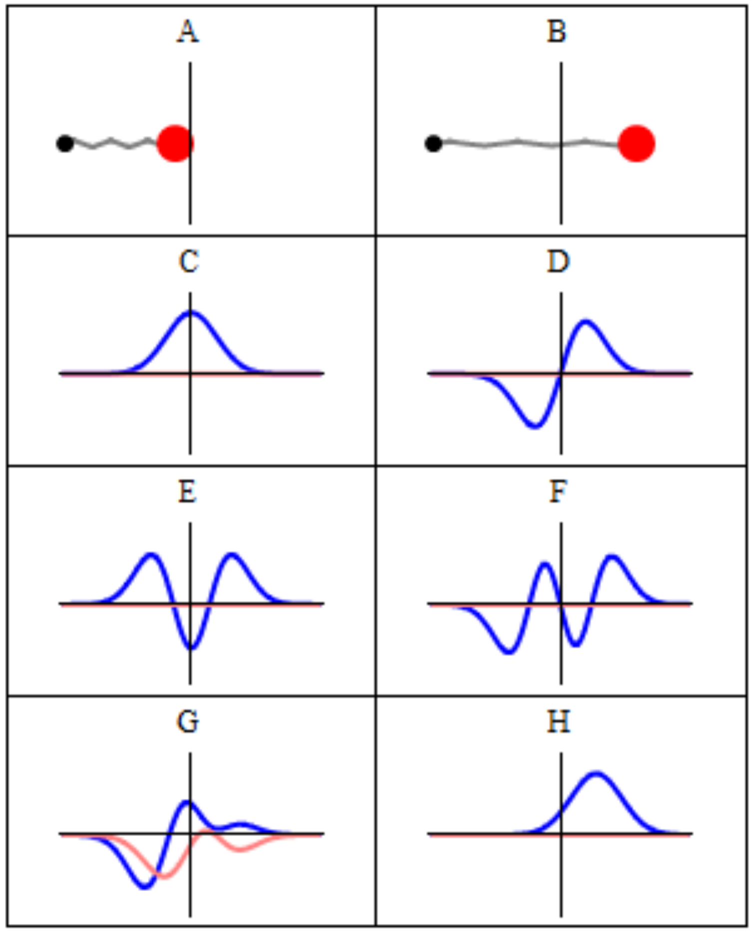
Quantum 1D harmonic oscillator

(Source: Wikipedia)

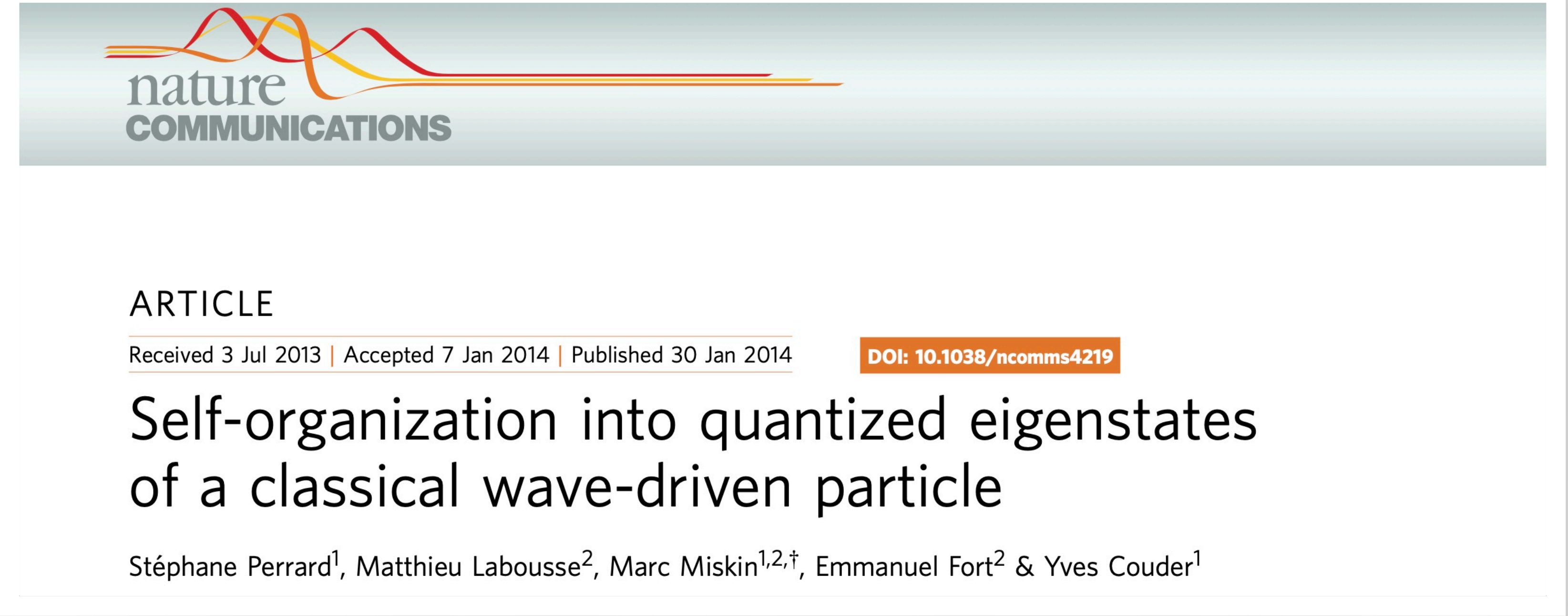
$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \qquad \hat{H}\left|\psi\right\rangle = E\left|\psi\right\rangle$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n\, n!}} \cdot \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right), \qquad n = 0, 1, 2, \ldots .$$

$$H_n(z) = (-1)^n\, e^{z^2} \frac{d^n}{dz^n}\left(e^{-z^2}\right) \qquad E_n = \hbar\omega\left(n + \tfrac{1}{2}\right) = (2n+1)\frac{\hbar}{2}\omega$$



Some trajectories of a harmonic oscillator according to Newton's laws of classical mechanics (A–B), and according to the Schrödinger equation of quantum mechanics (C–H). In A–B, the particle (represented as a ball attached to a spring) oscillates back and forth. In C–H, some solutions to the Schrödinger Equation are shown, where the horizontal axis is position, and the vertical axis is the real part (blue) or imaginary part (red) of the wavefunction. C, D, E, F, but not G, H, are energy eigenstates. H is a coherent state—a quantum state that approximates the classical trajectory.



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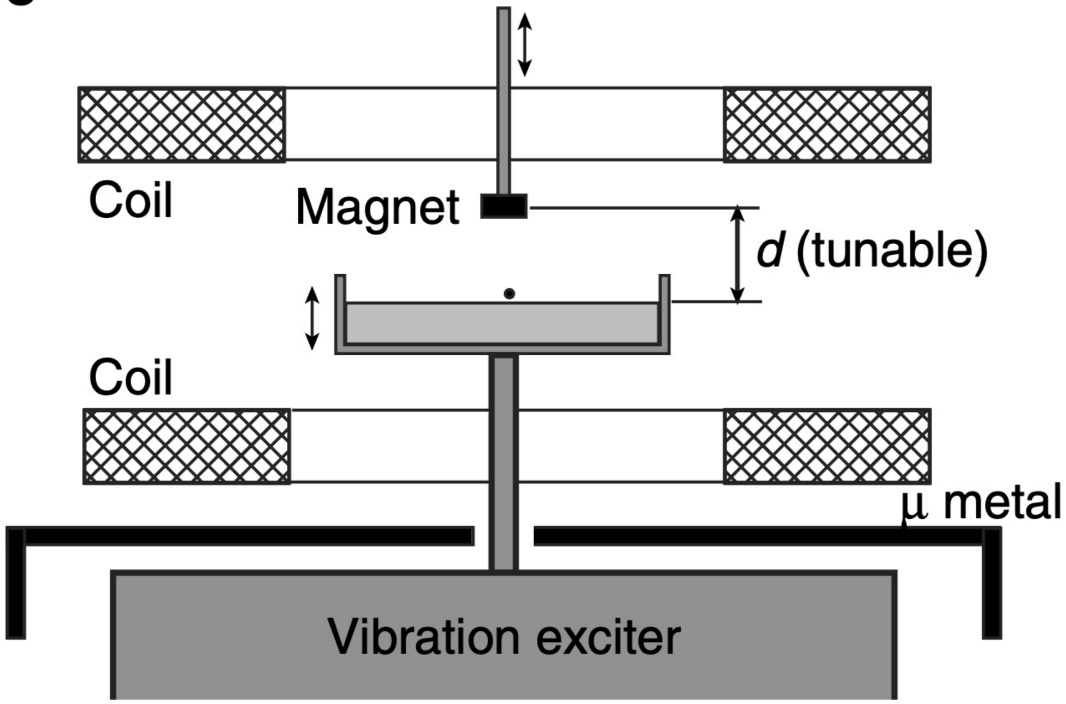
c

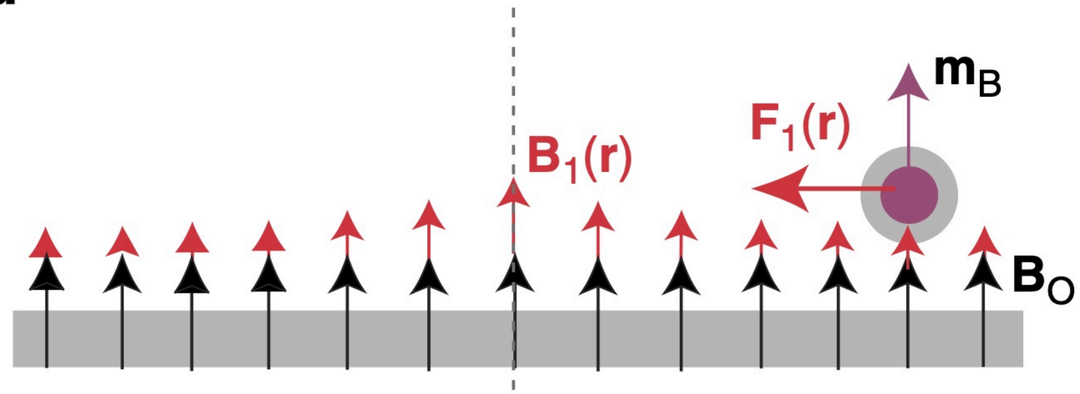


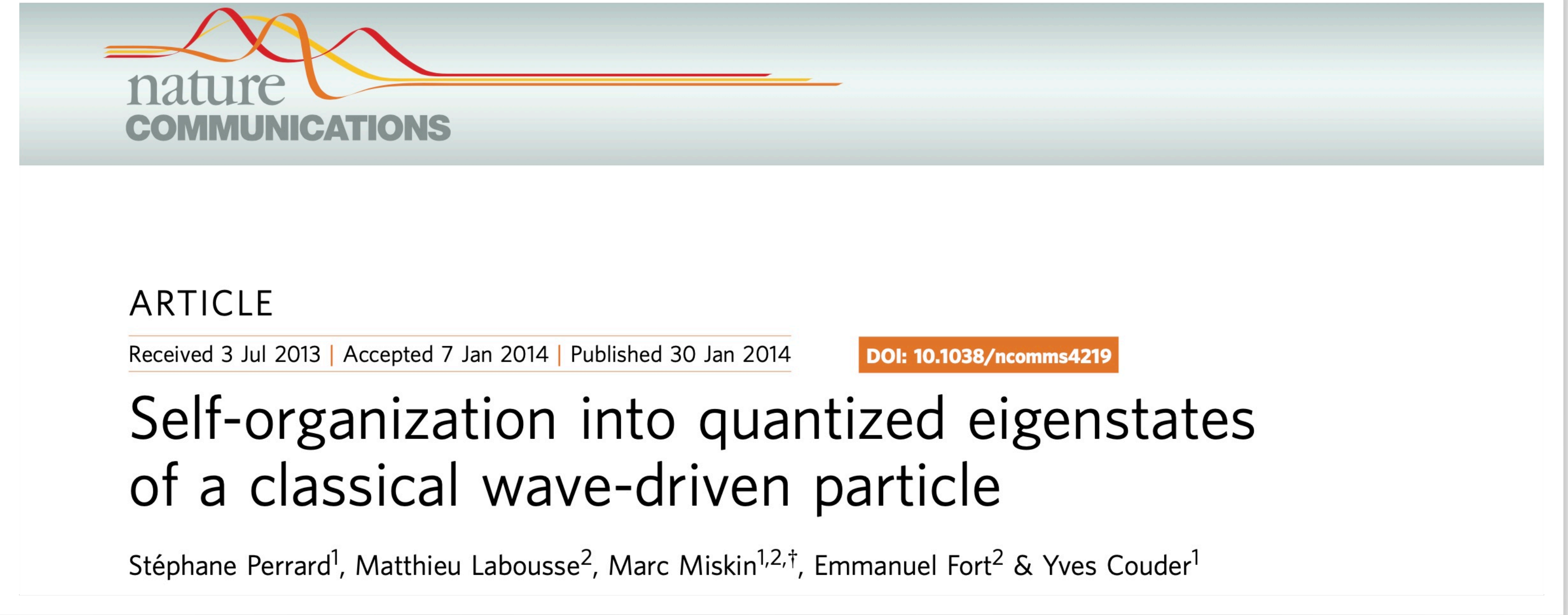
$$\mathbf{F}_{\mathrm{m}}(d) = -\kappa(d)\mathbf{r}$$

$$\omega = \sqrt{\kappa(d)/m_{\mathrm{W}}}$$

where m_{W} is the effective mass of the walker.

d





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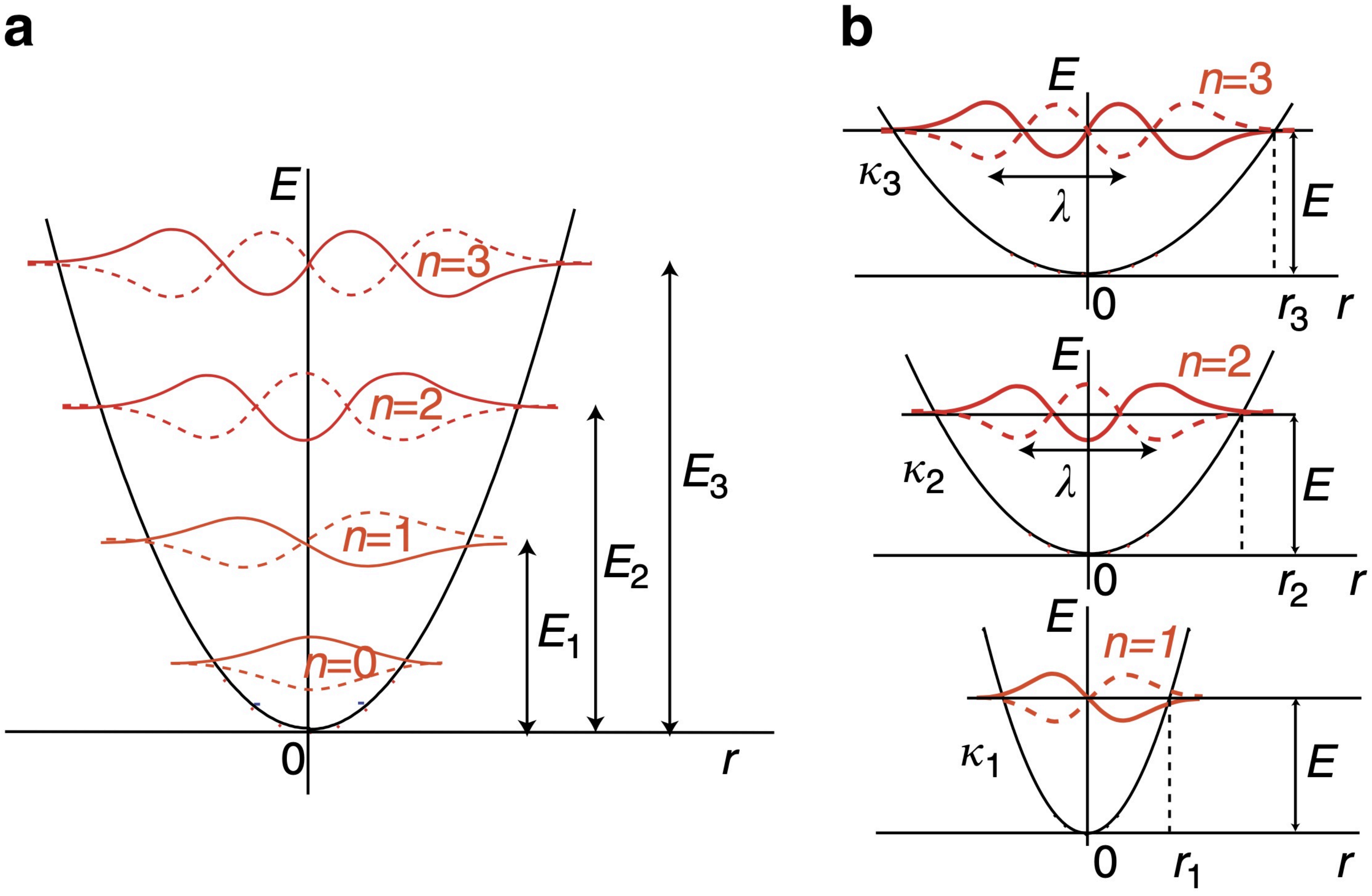

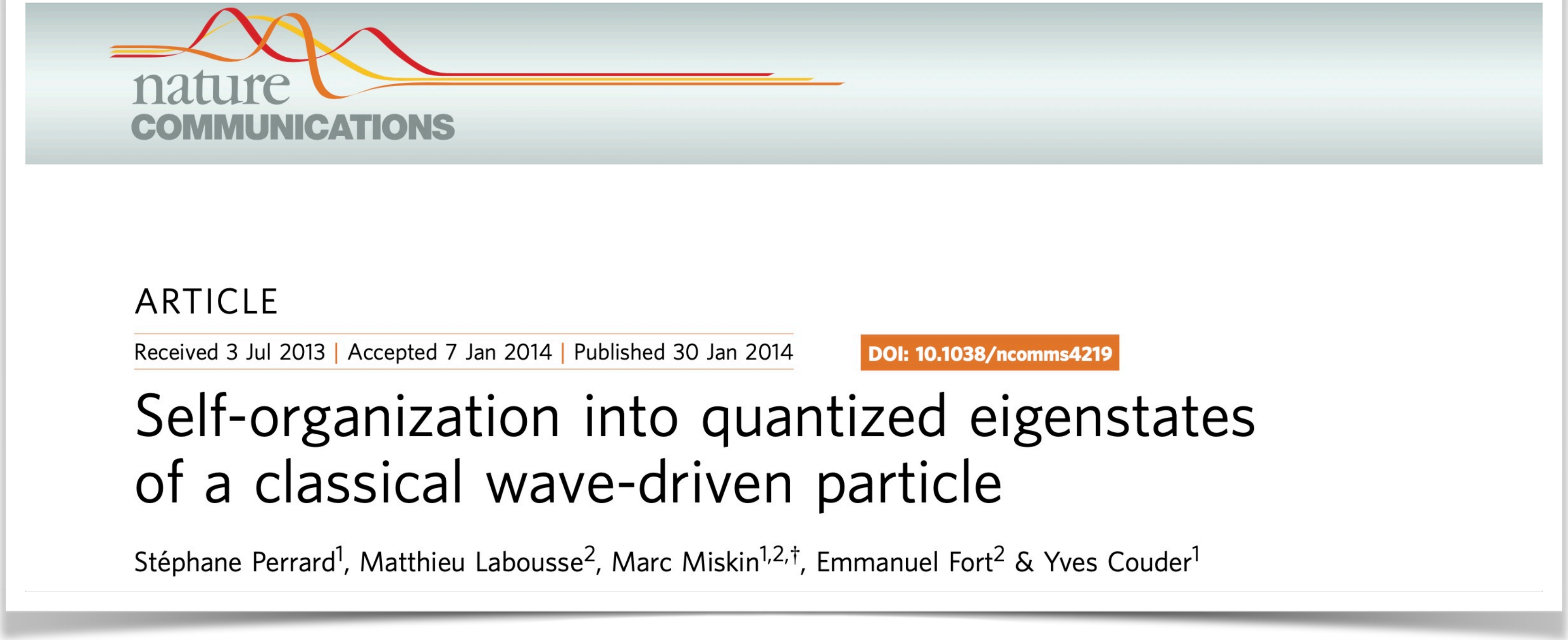

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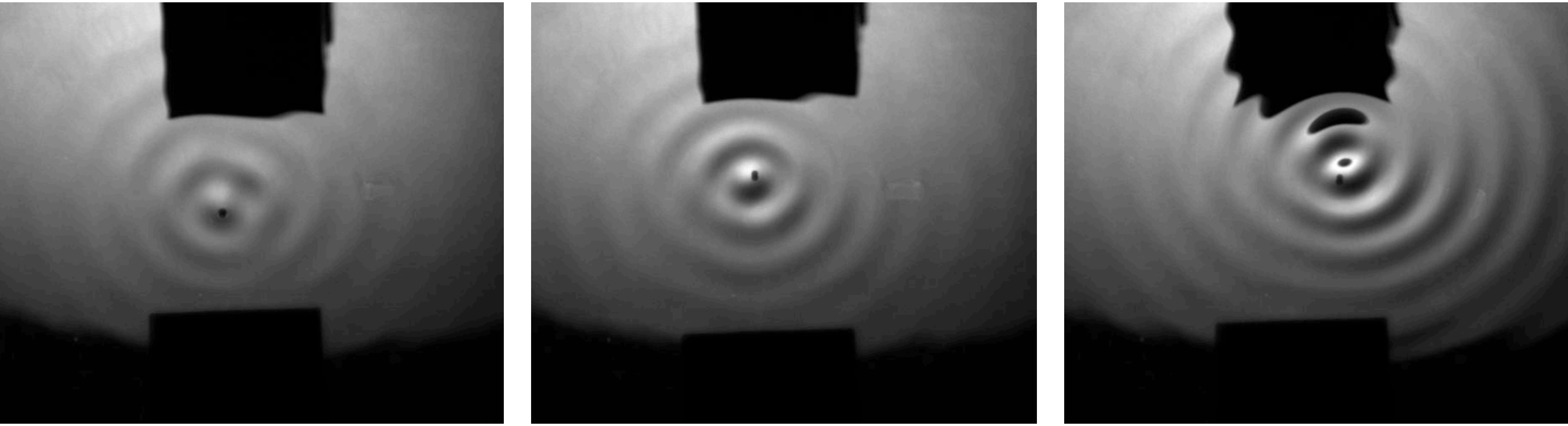

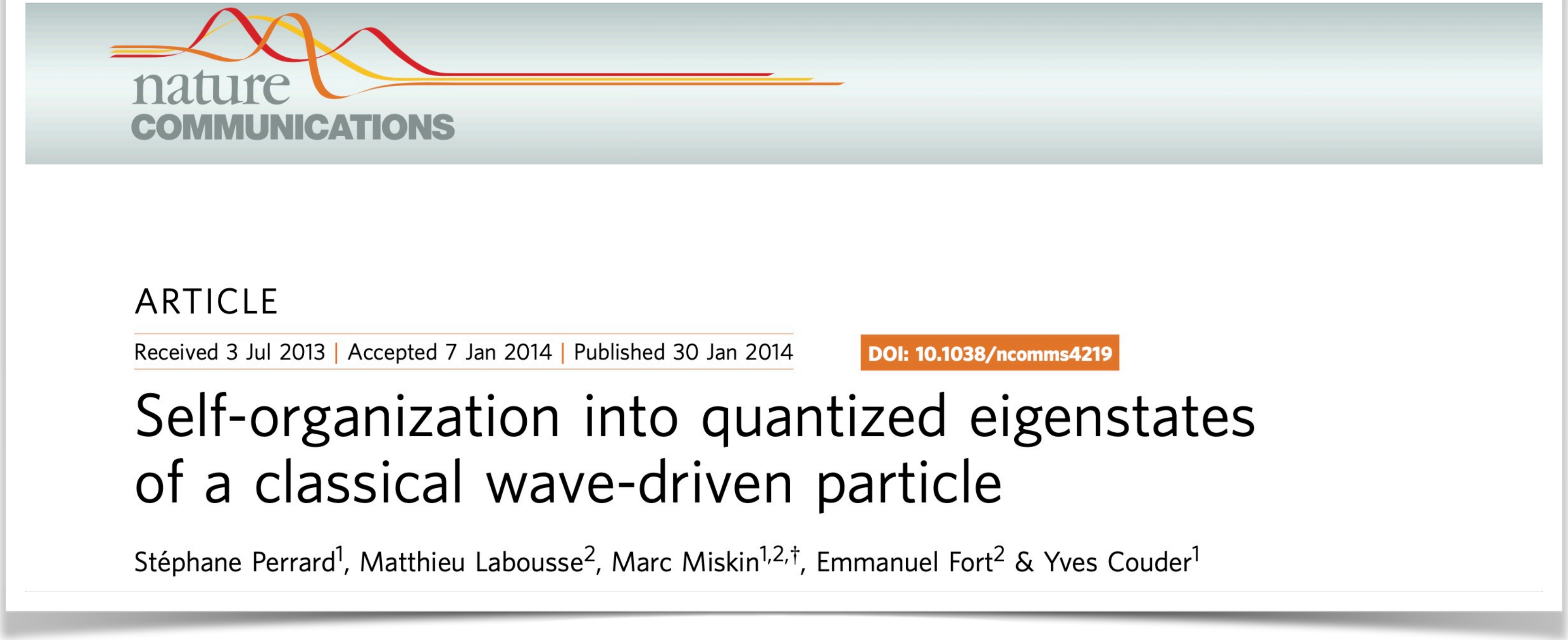

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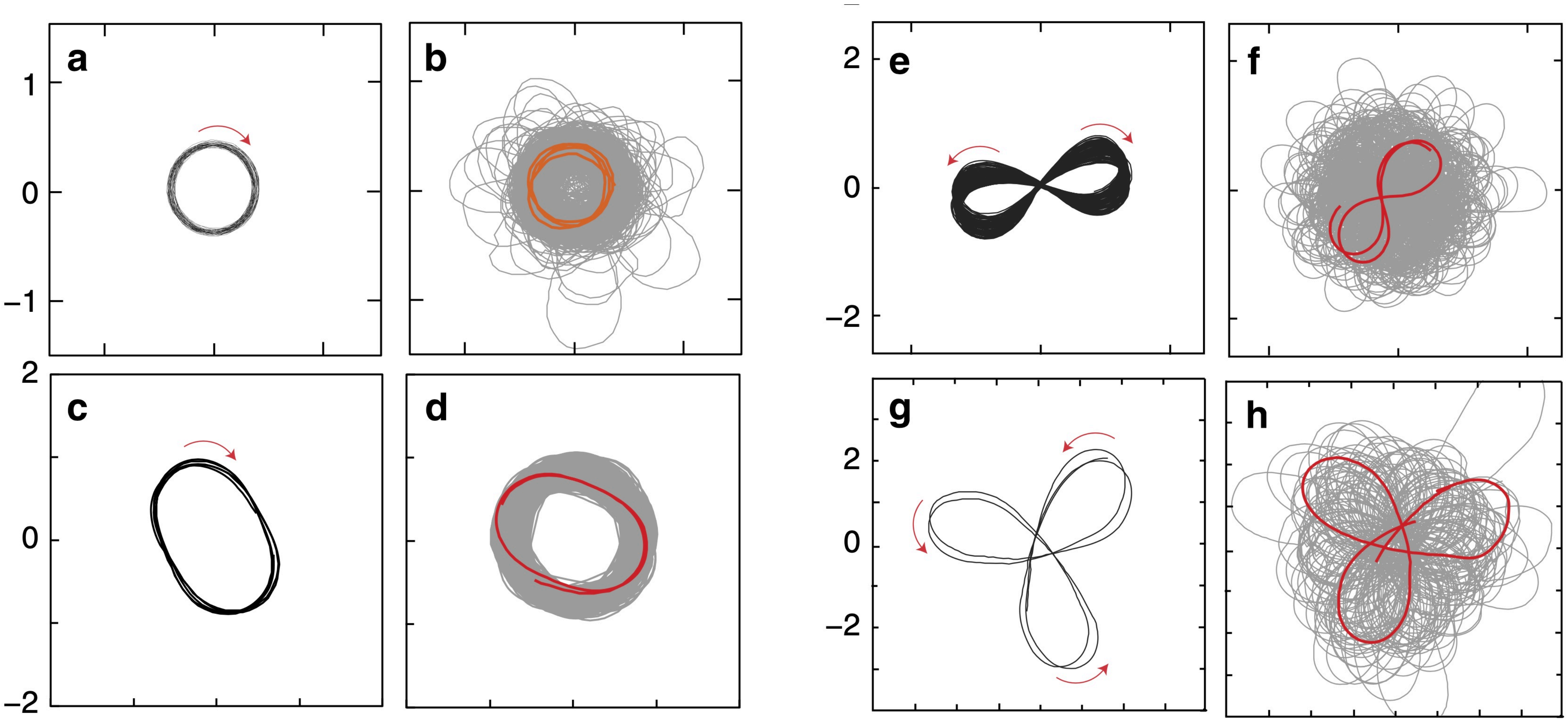

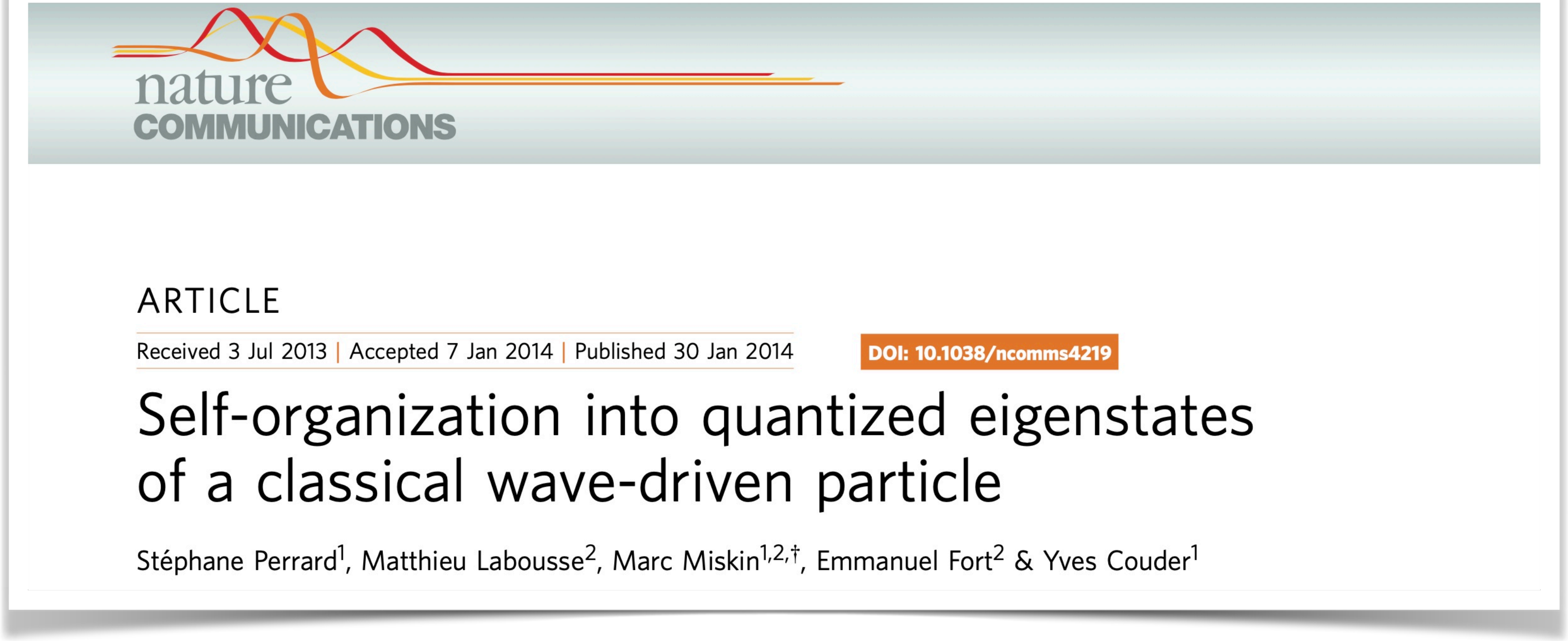
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A double quantization. We characterize the particle's motion by two features, its mean non-dimensional distance to the central axis $\bar{R}$:

$$\bar{R} = \frac{\sqrt{\langle R^2 \rangle}}{\lambda_F} = \frac{1}{N}\sqrt{\sum_{k=1}^{N} \frac{r_k^2(t)}{\lambda_F^2}}. \tag{3}$$

and its mean non-dimensional angular momentum $\bar{L}_z$:

$$\bar{L}_z = \frac{\langle L_z \rangle}{m_W \lambda_F V} = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{\mathbf{r}_k}{\lambda_F} \times \frac{\mathbf{V}_k}{V}\right). \tag{4}$$

where r_k is the position of the kth bounce and N is the total number of bounces.

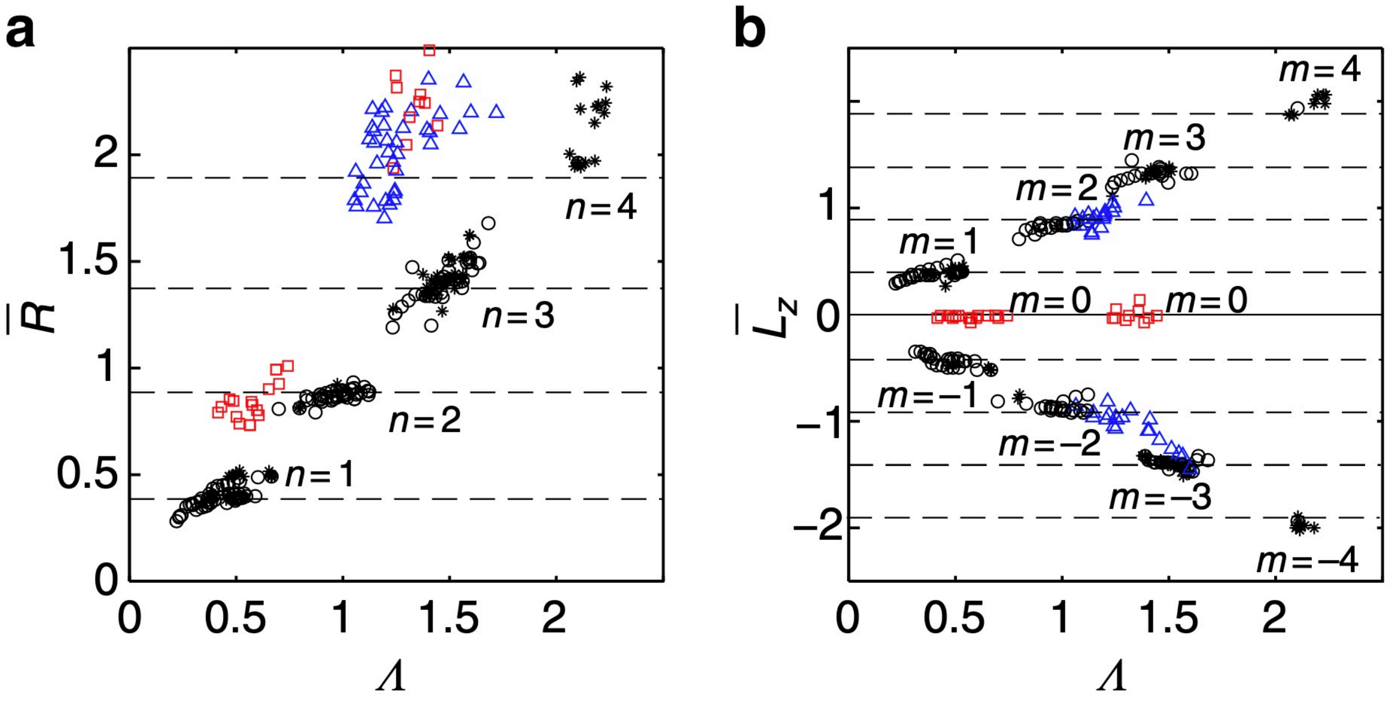
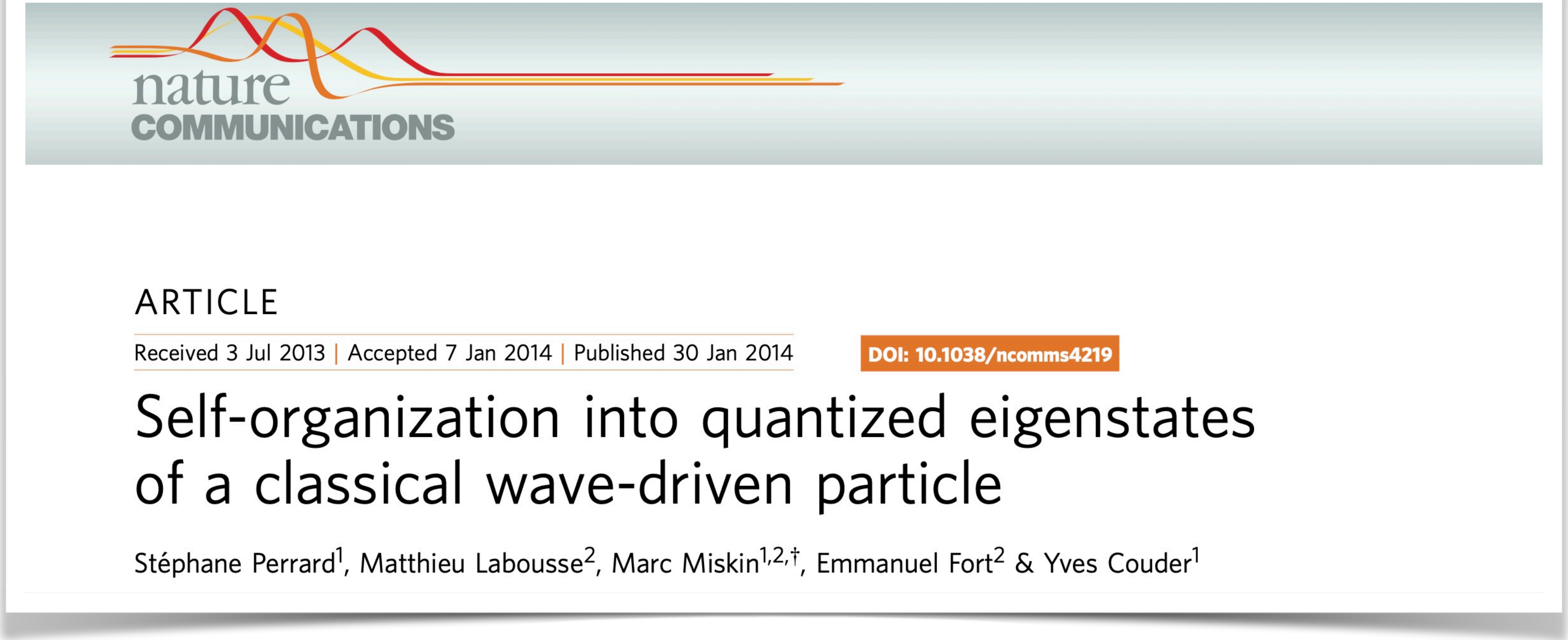


Figure 4 | The double quantization at long memory. (**a**) Evolution of the mean spatial extent $\bar{R}$ as a function of the control parameter Λ. The black circles correspond to the circular or oval orbits, the red squares represent the lemniscates and the blue triangles represent the trefoils. The stars are radii measured in organized sections of disordered patterns of the type shown below in Fig. 5. (**b**) Evolution of the mean angular momentum $\bar{L}_z$ as a function of Λ for the same set of trajectories.

dimensionless width of the potential well $\Lambda = (V\sqrt{m_W/\kappa})/\lambda_F$



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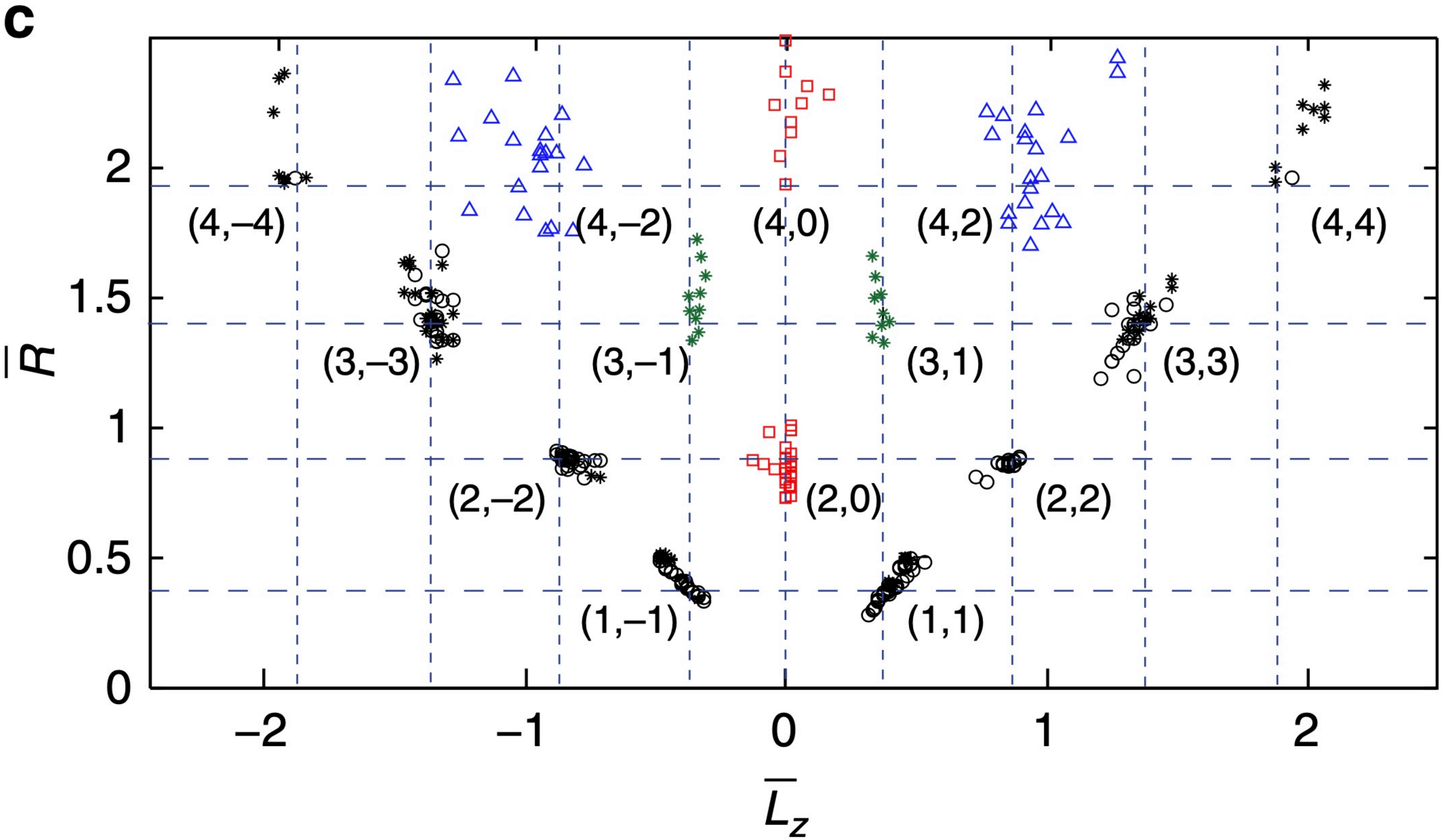
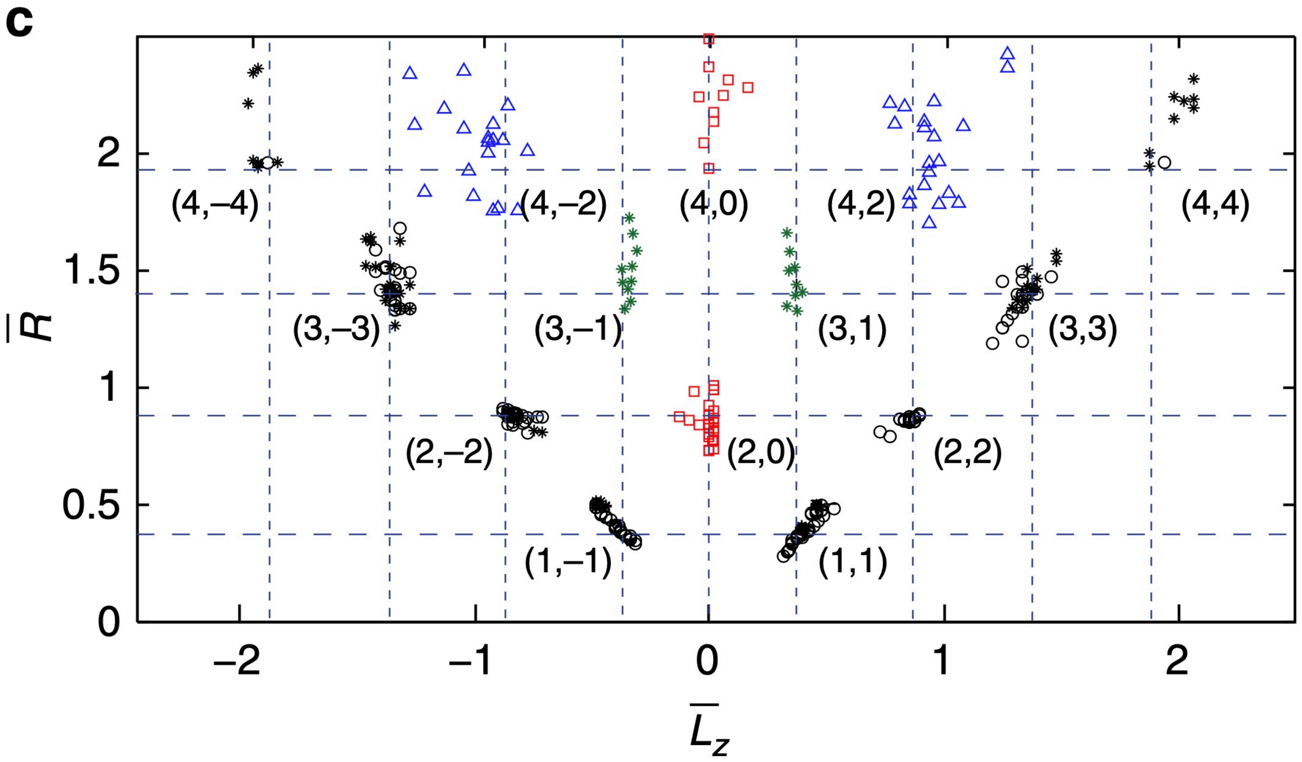




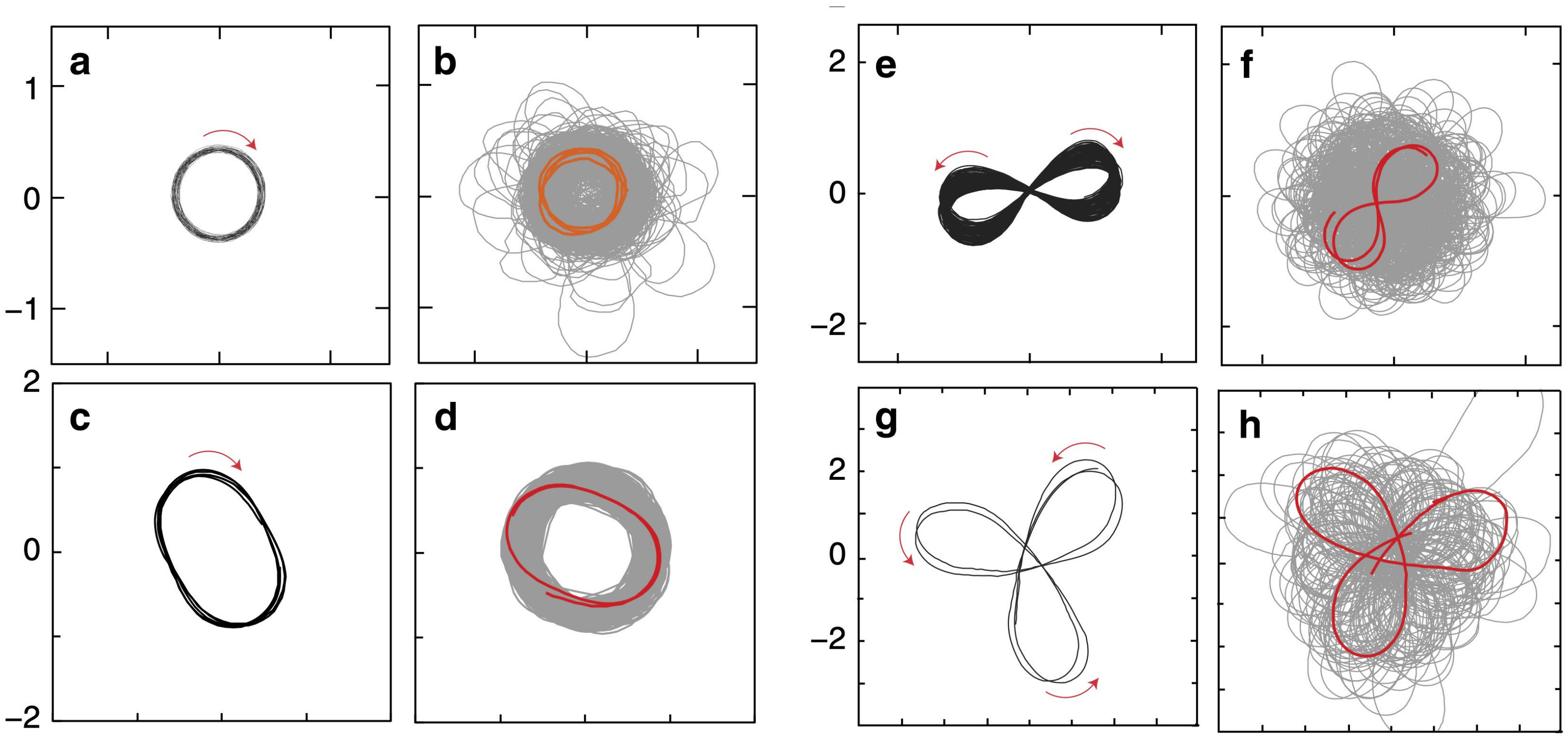
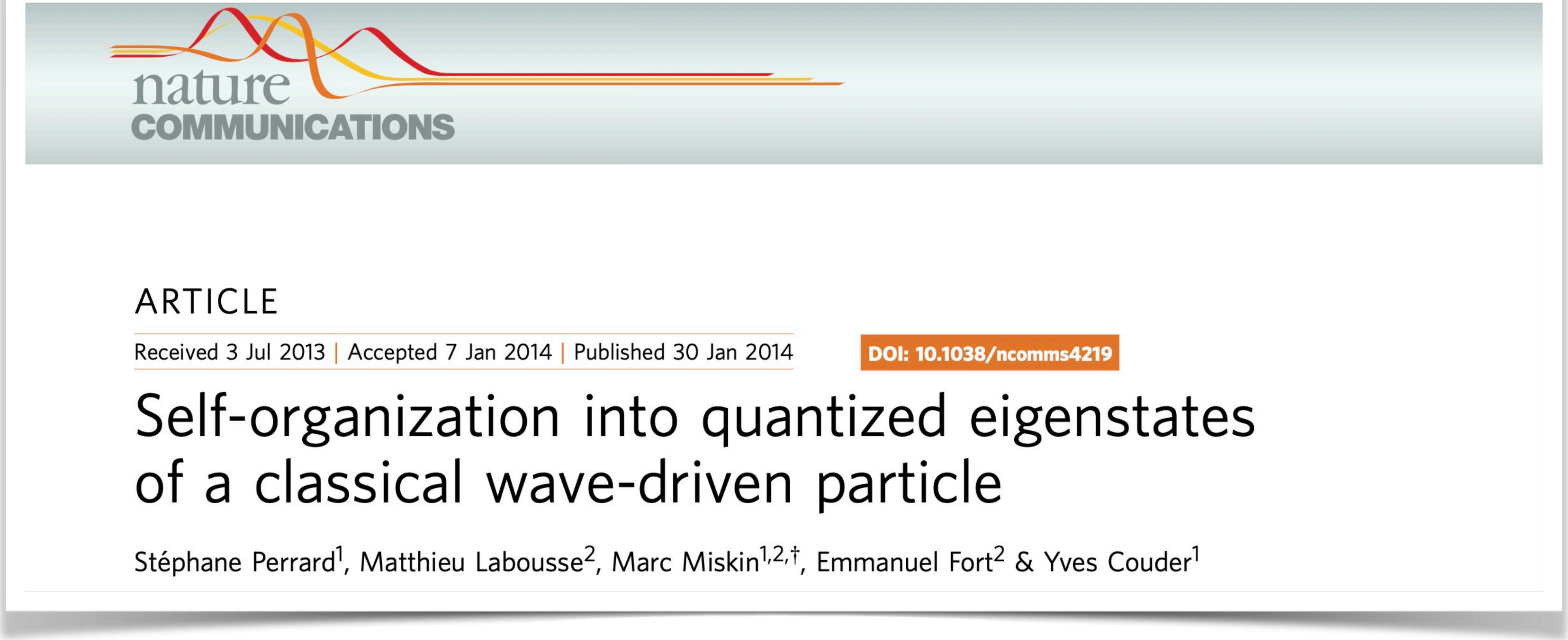


Figure 3 | Four of the experimentally observed long memory stable orbits. Simple trajectories can be observed in narrow ranges of values of the tuning parameter Λ. They are shown in their most stable form (for $M \approx 50$) and in situations where they are slightly unstable (for $M \approx 100$). The scales are in units of the Faraday wavelength $\lambda_F = 4.75$ mm. (**a**,**b**) The smallest circular orbit ($n = 1$, $m = \pm 1$). (**c**,**d**) The oval orbit ($n = 2$, $m = \pm 2$) well fitted by a Cassini oval. (**e**,**f**) The small lemniscates ($n = 2$, $m = 0$). (**g**,**h**) The trefoil ($n = 4$, $m = \pm 2$). In **b**,**d**,**f**,**h** the trajectories are shown in grey for a long time interval and a part of it has been singled out in red. The scales are in units of λ_F. See Supplementary Movies 1–3 associated with trajectories of **a**,**d**,**f**, respectively.



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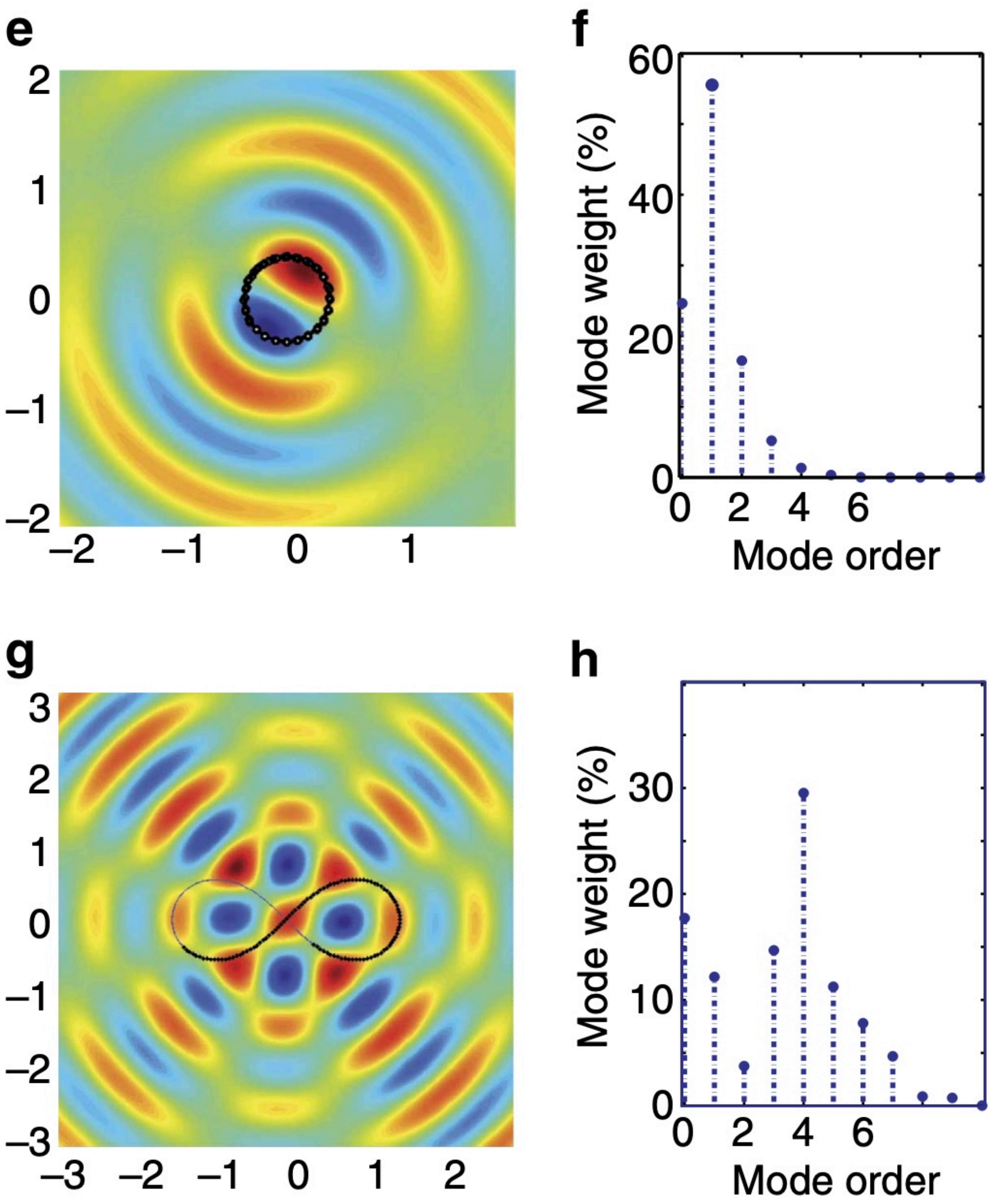
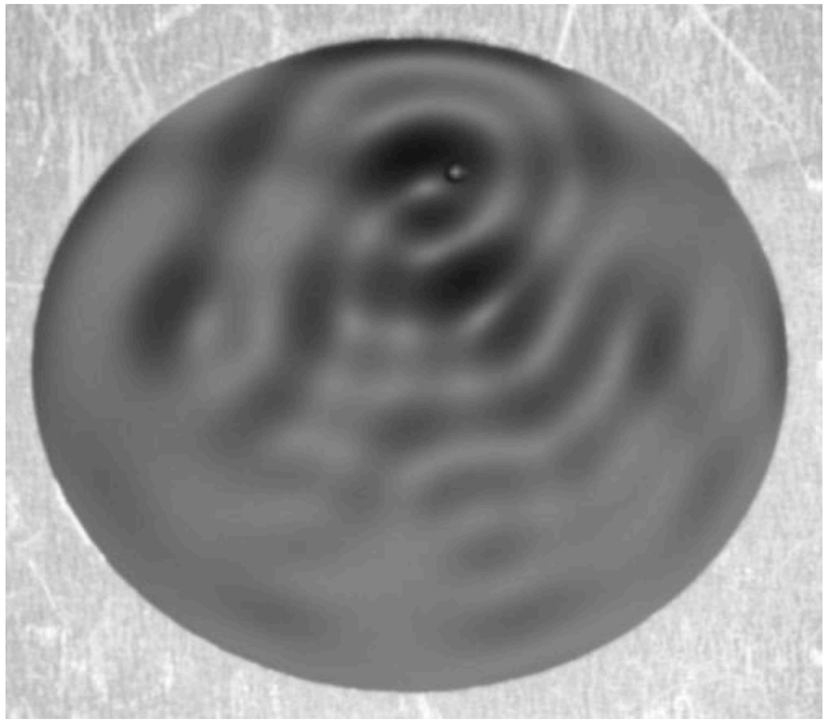


(e) The experimental trajectory of an experimental circular orbit ($n=1$, $m=1$) at a memory parameter $M=32$. Scales are in λ_F units. **(f)** Spectral decomposition of the wave field in centred Bessel functions. The trajectory being close to the ideal, the amplitude of the J_0 mode is weak, the dominant J_1 mode is responsible for the azimuthal propulsion of the droplet. **(g)** An experimentally observed lemniscate trajectory ($n=2$, $m=0$). The latest M impacts are shown as open dots. It is superimposed on the reconstructed global wave field. **(h)** The spectral decomposition of this wave field showing that for this near-ideal orbit the J_2 mode is specifically weak.

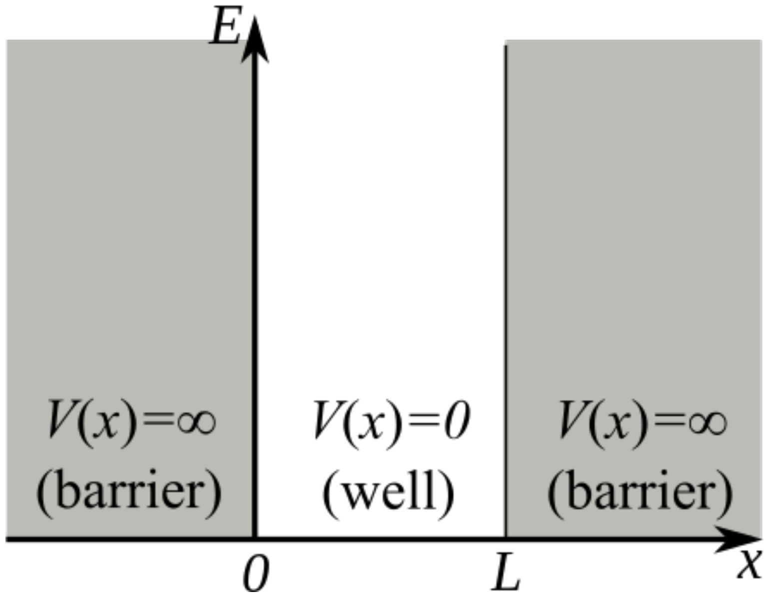
Wavelike statistics in cavities



Particle in a box, or infinite potential well

Source: Wikipedia

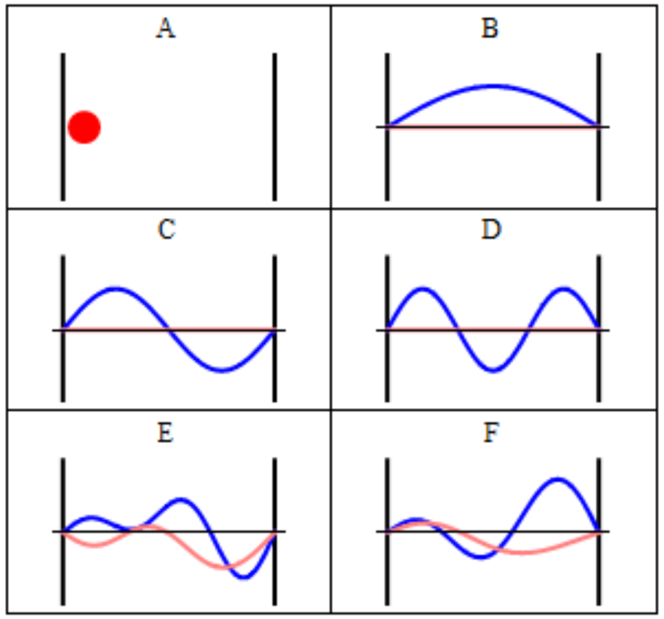
$$V(x) = \begin{cases} 0, & x_c - \frac{L}{2} < x < x_c + \frac{L}{2}, \\ \infty, & \text{otherwise,} \end{cases}$$



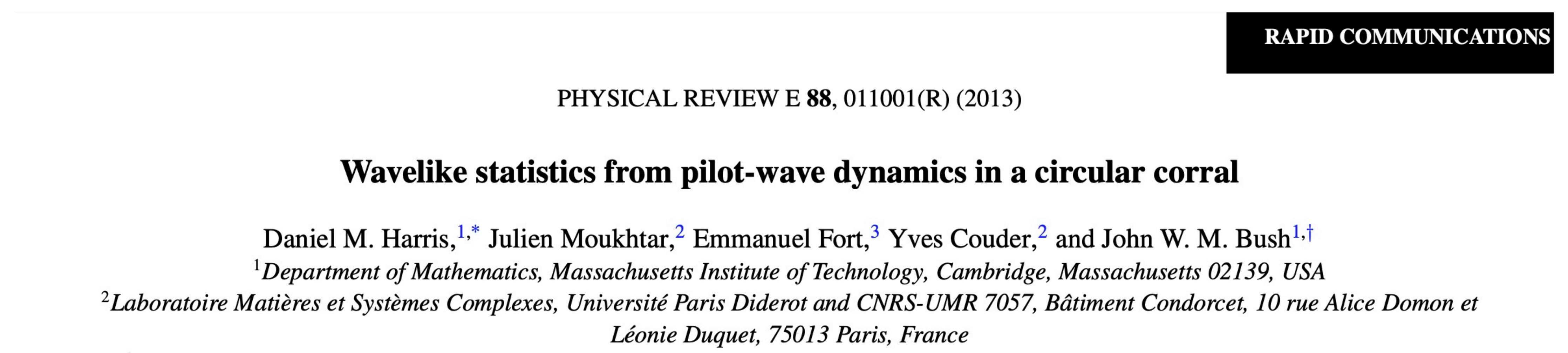
$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t)$$

$$\psi_n(x,t) = \begin{cases} A\sin\left(k_n\left(x - x_c + \frac{L}{2}\right)\right)e^{-i\omega_n t} & x_c - \frac{L}{2} < x < x_c + \frac{L}{2}, \\ 0 & \text{otherwise} \end{cases}$$

$$k_n = \frac{n\pi}{L} \qquad E_n = \hbar\omega_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$



The states (B,C,D) are energy eigenstates, but (E,F) are not.



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[2]*Laboratoire Matières et Systèmes Complexes, Université Paris Diderot and CNRS-UMR 7057, Bâtiment Condorcet, 10 rue Alice Domon et Léonie Duquet, 75013 Paris, France*
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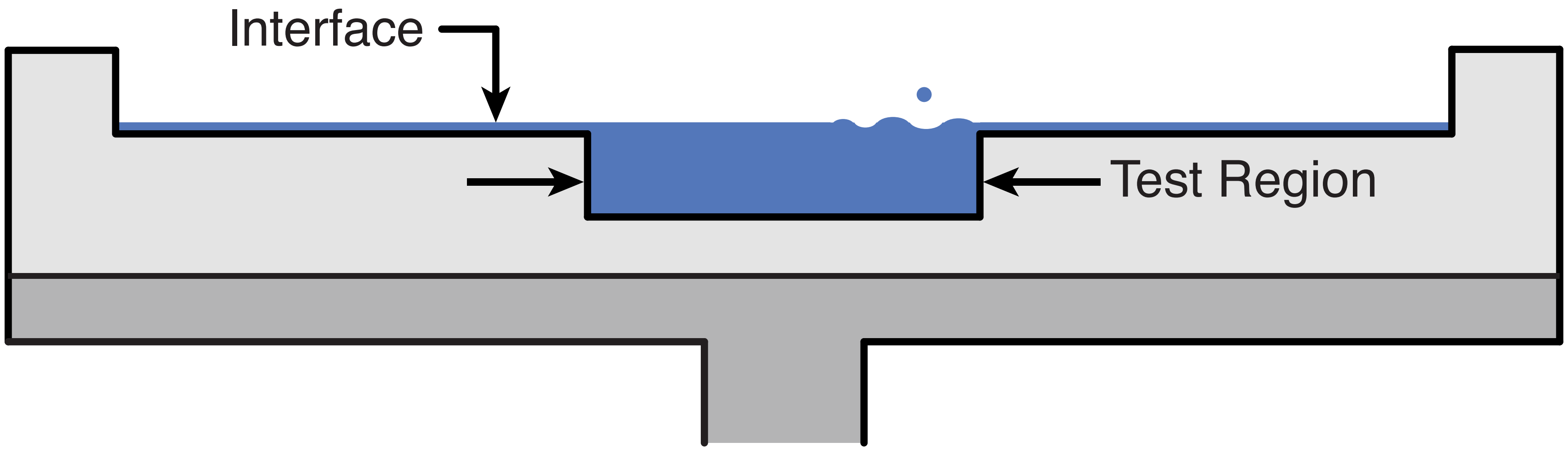
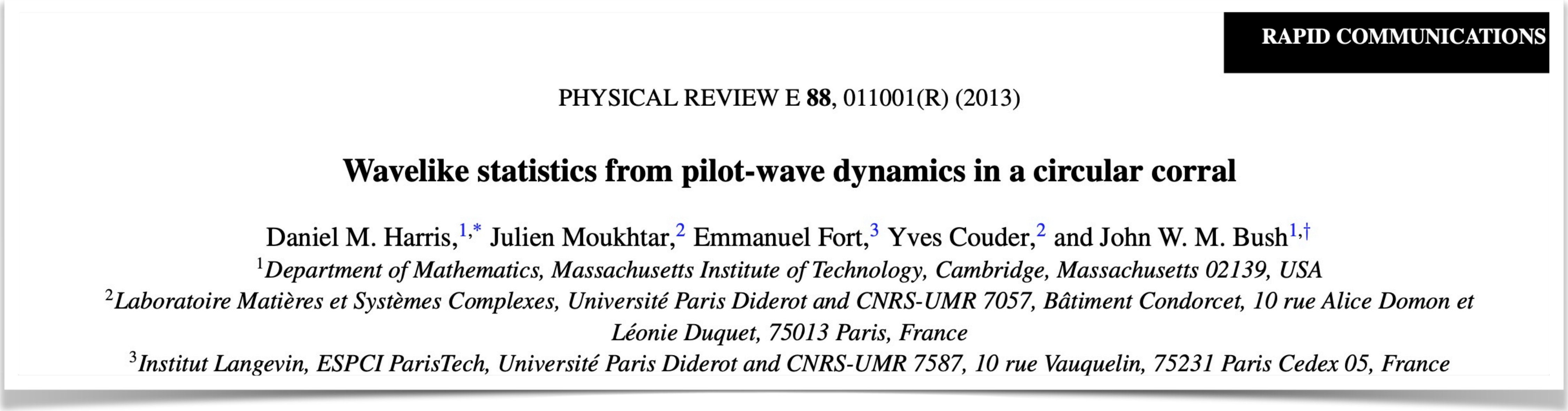
Walker in a circular corral



Droplet can only walk in the deep central region,
thus remains confined within the “corral”



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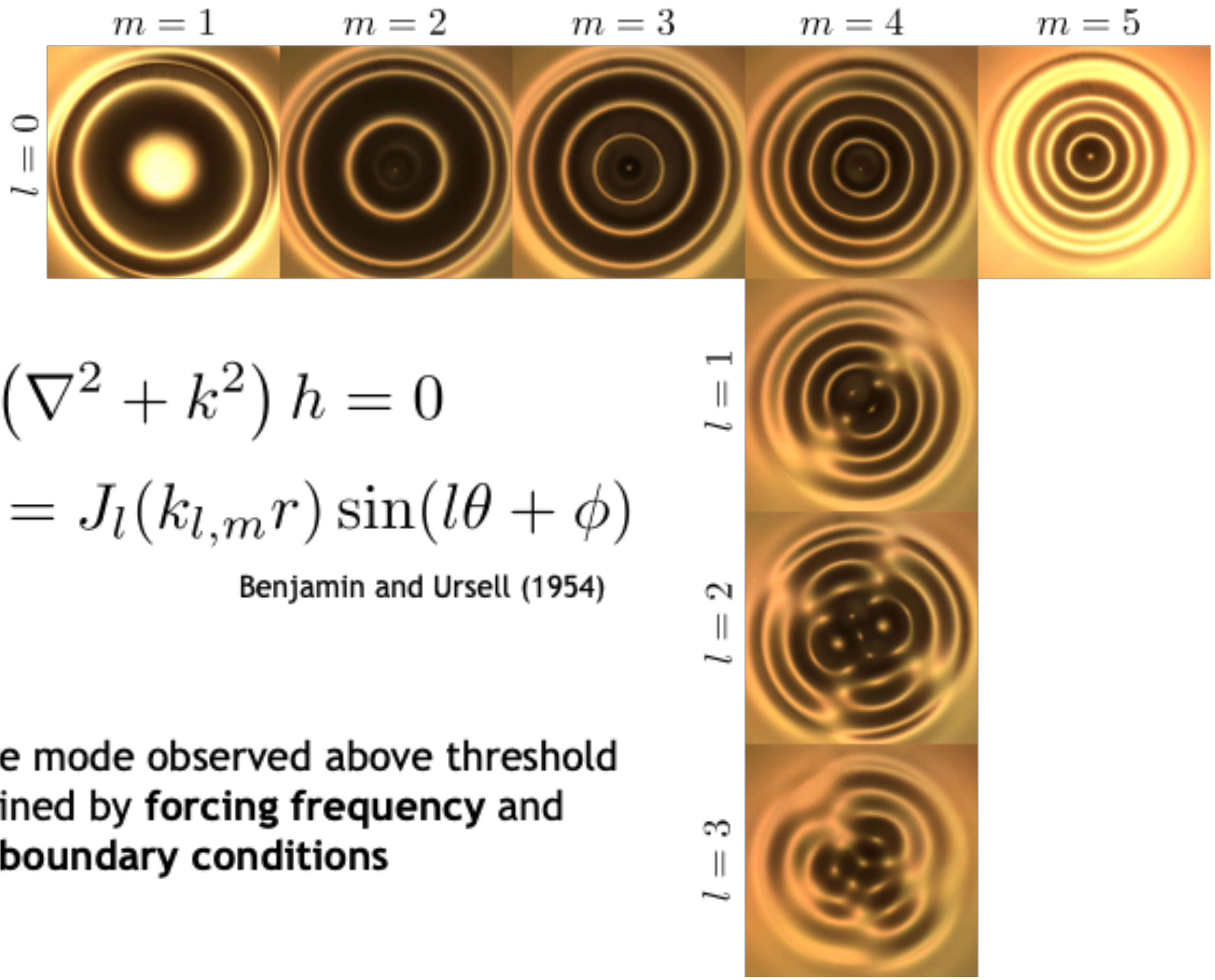
Circular Faraday modes

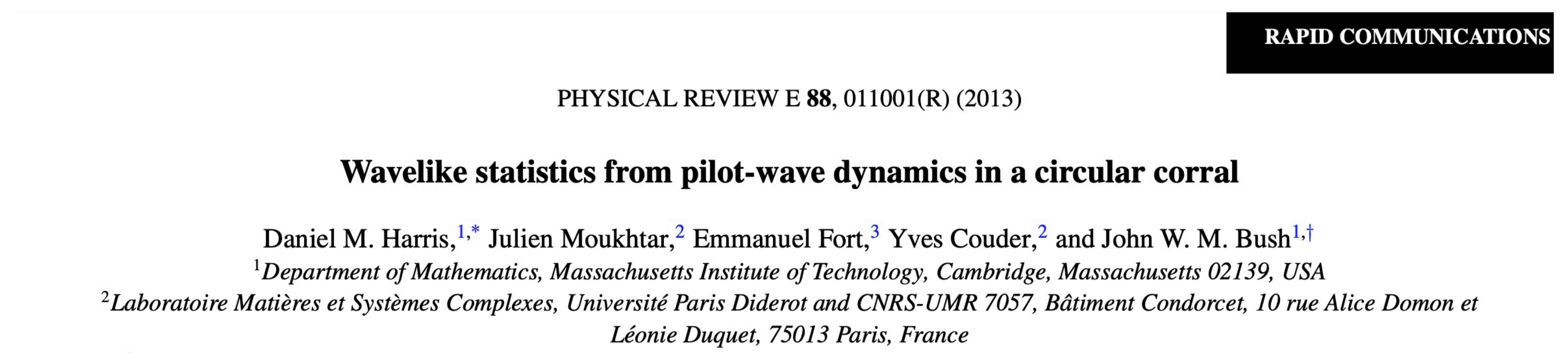


$$\left(\nabla^2 + k^2\right) h = 0$$

$$h_{l,m} = J_l(k_{l,m} r) \sin(l\theta + \phi)$$

Benjamin and Ursell (1954)

Faraday wave mode observed above threshold
is determined by **forcing frequency** and
boundary conditions



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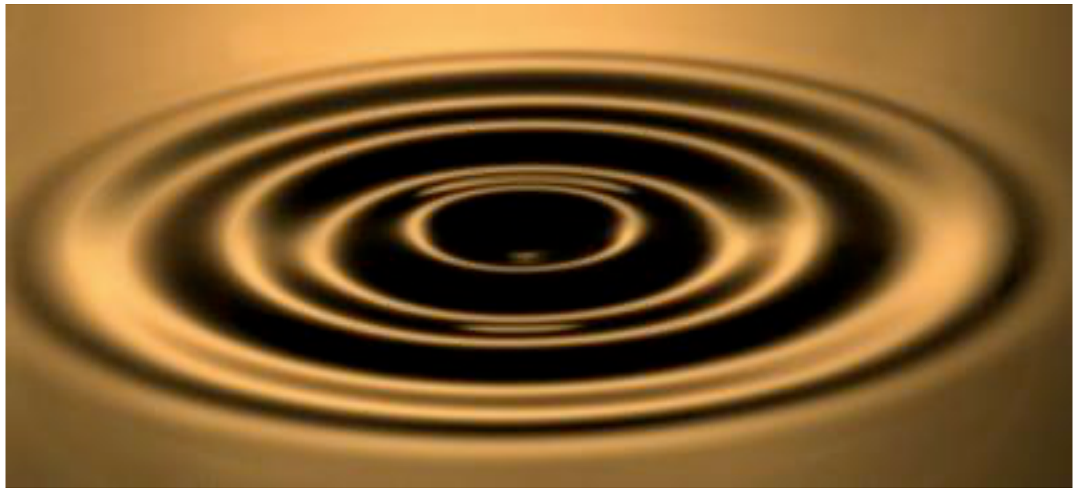
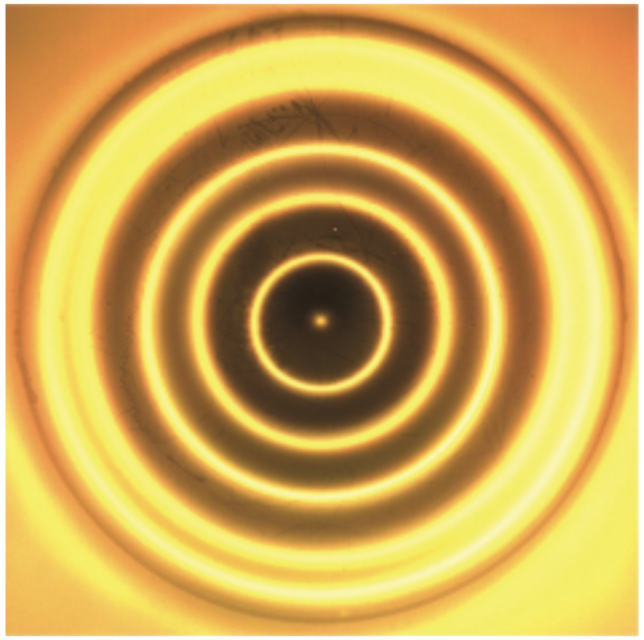
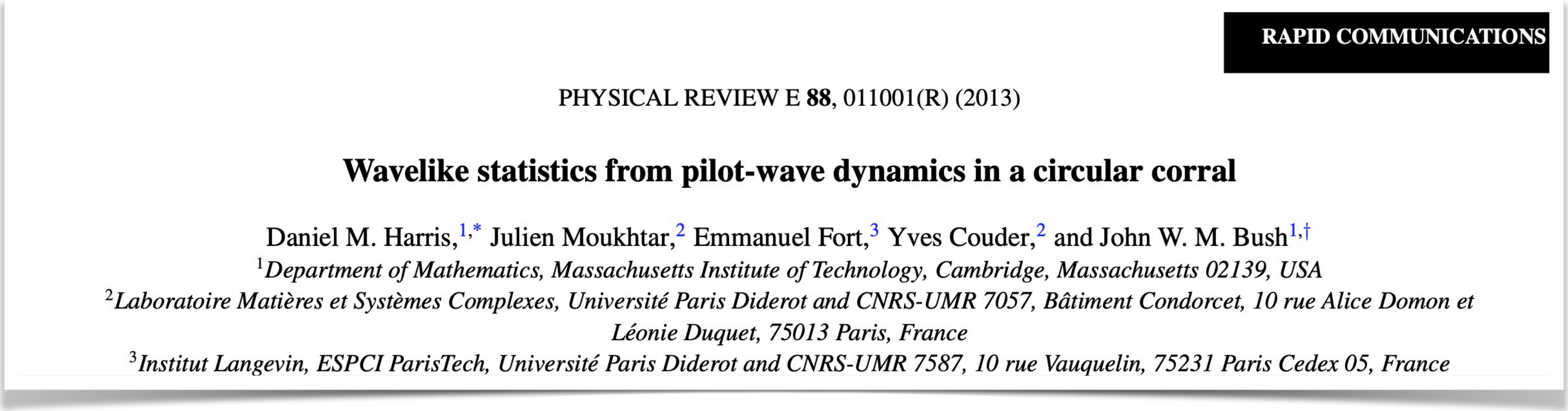
Selected Faraday mode





$m = 5,\ l = 0$

Most unstable mode above the Faraday threshold at $f = 70\ \mathrm{Hz}$



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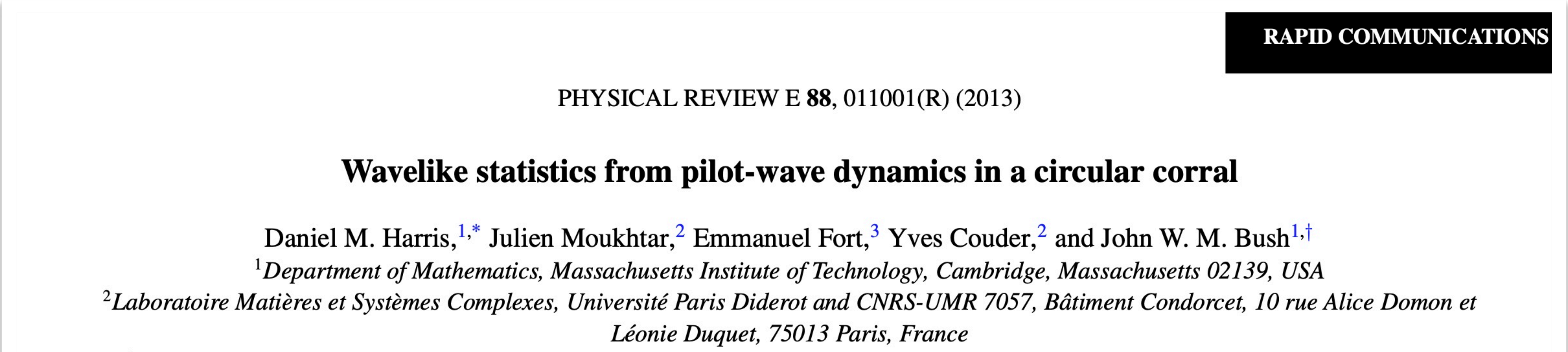
Walker in circular corral

- Without the presence of the drop, the surface remains flat



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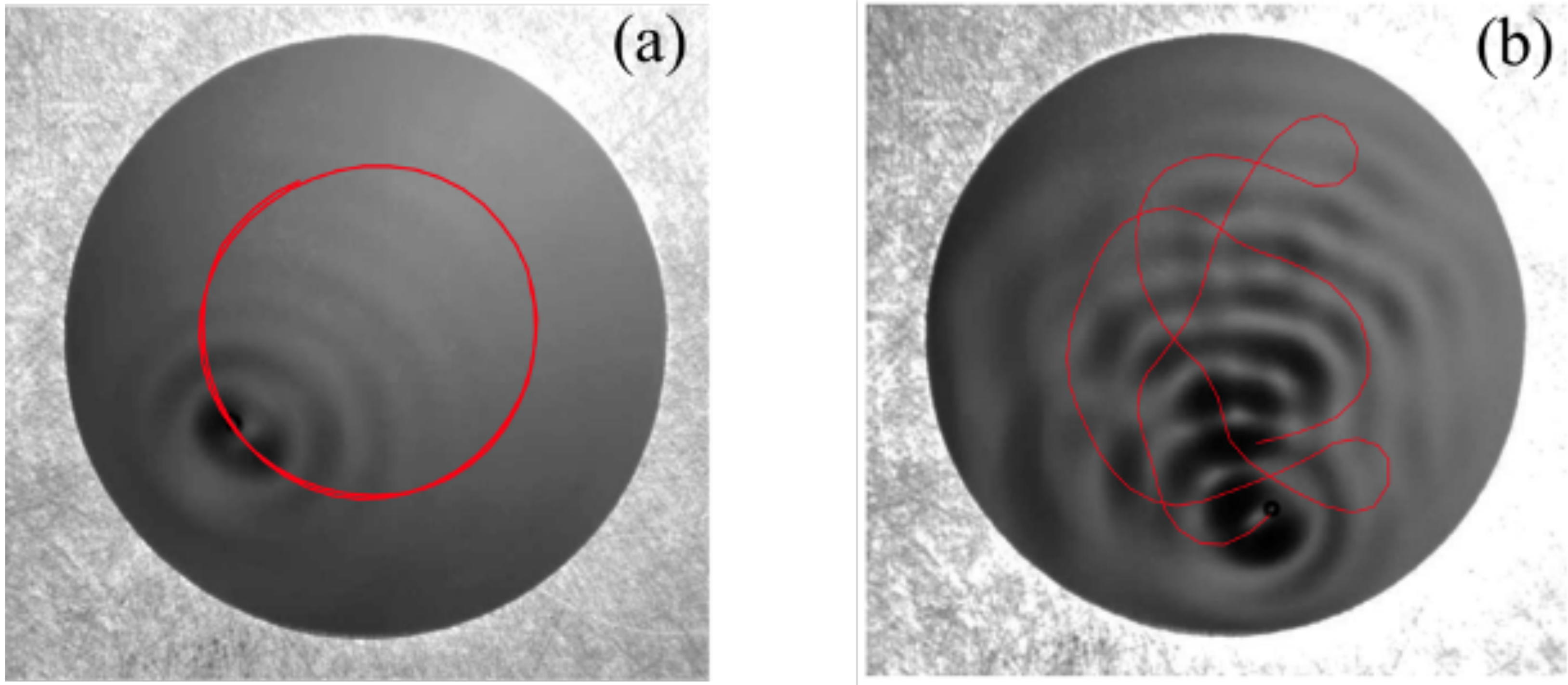
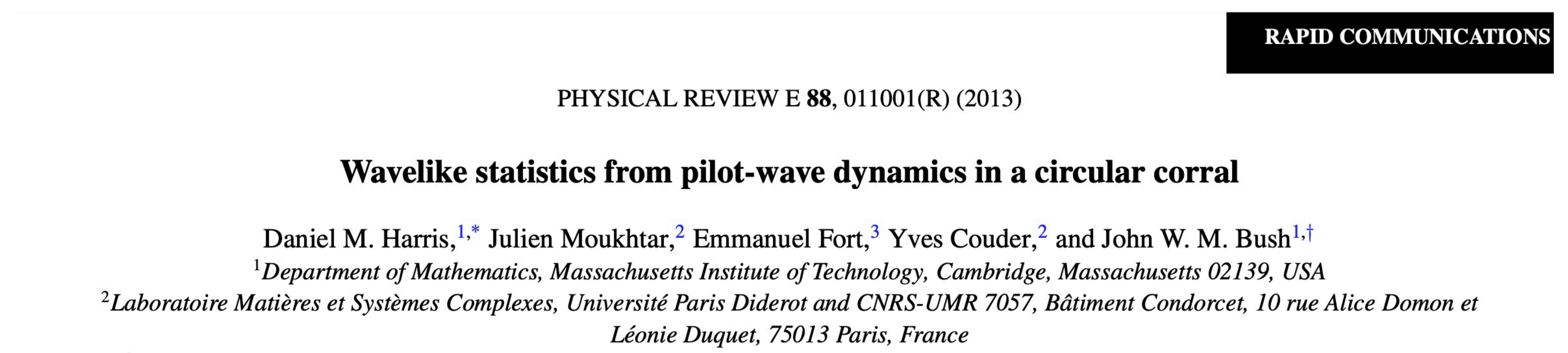
Influence of memory

Increased forcing amplitude



- As the Faraday threshold is approached, trajectory becomes irregular



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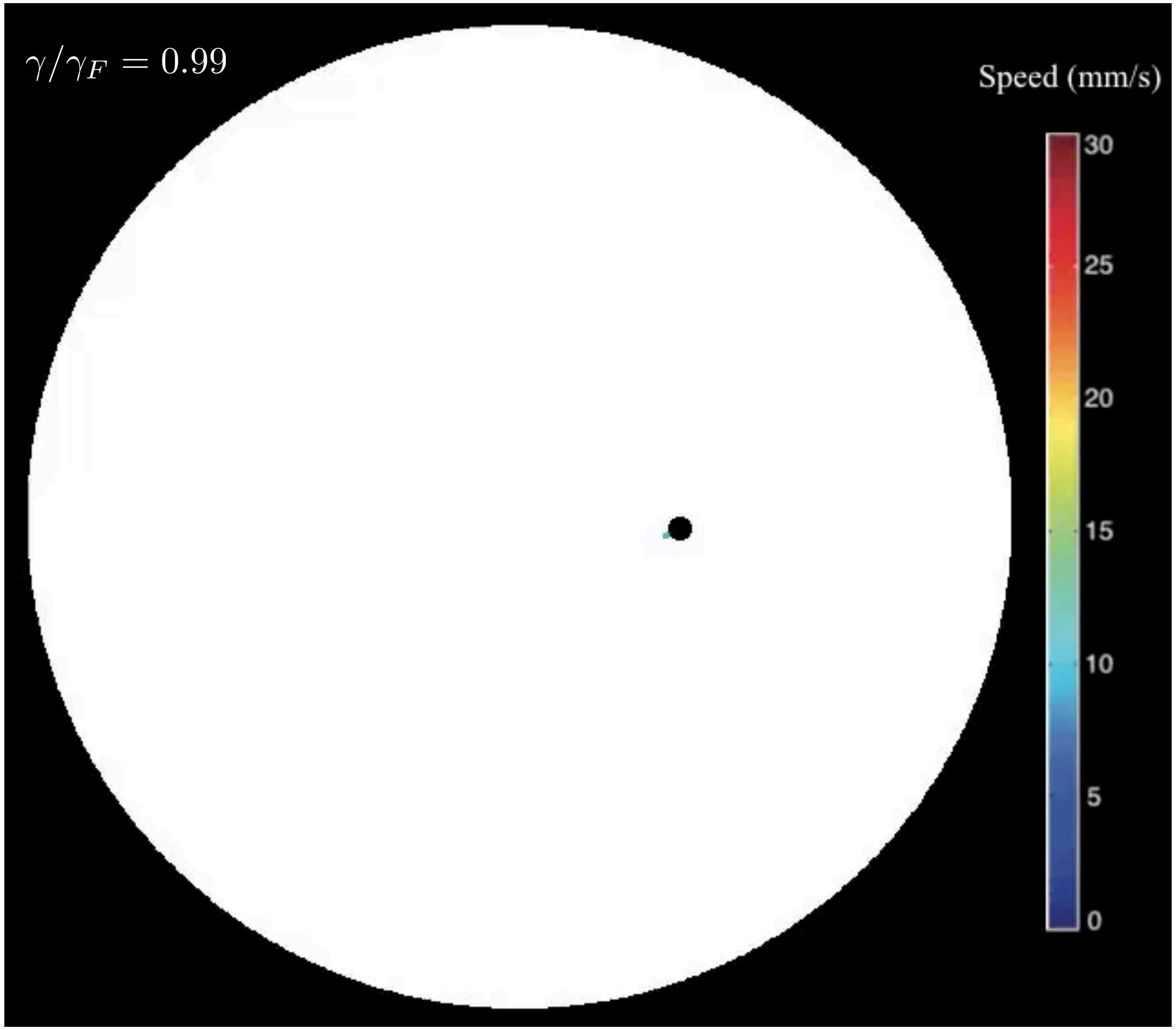

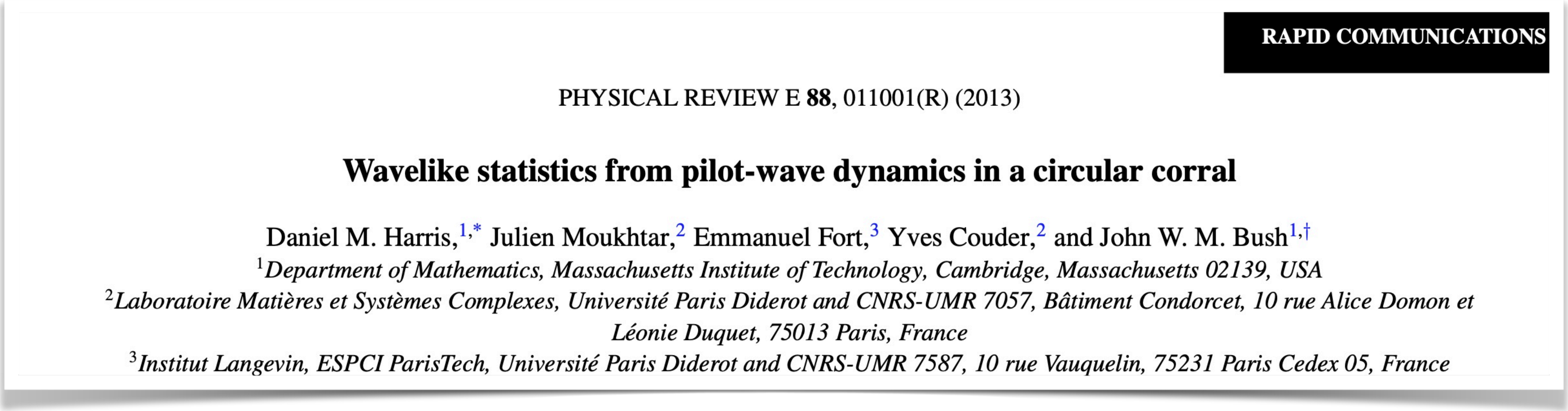

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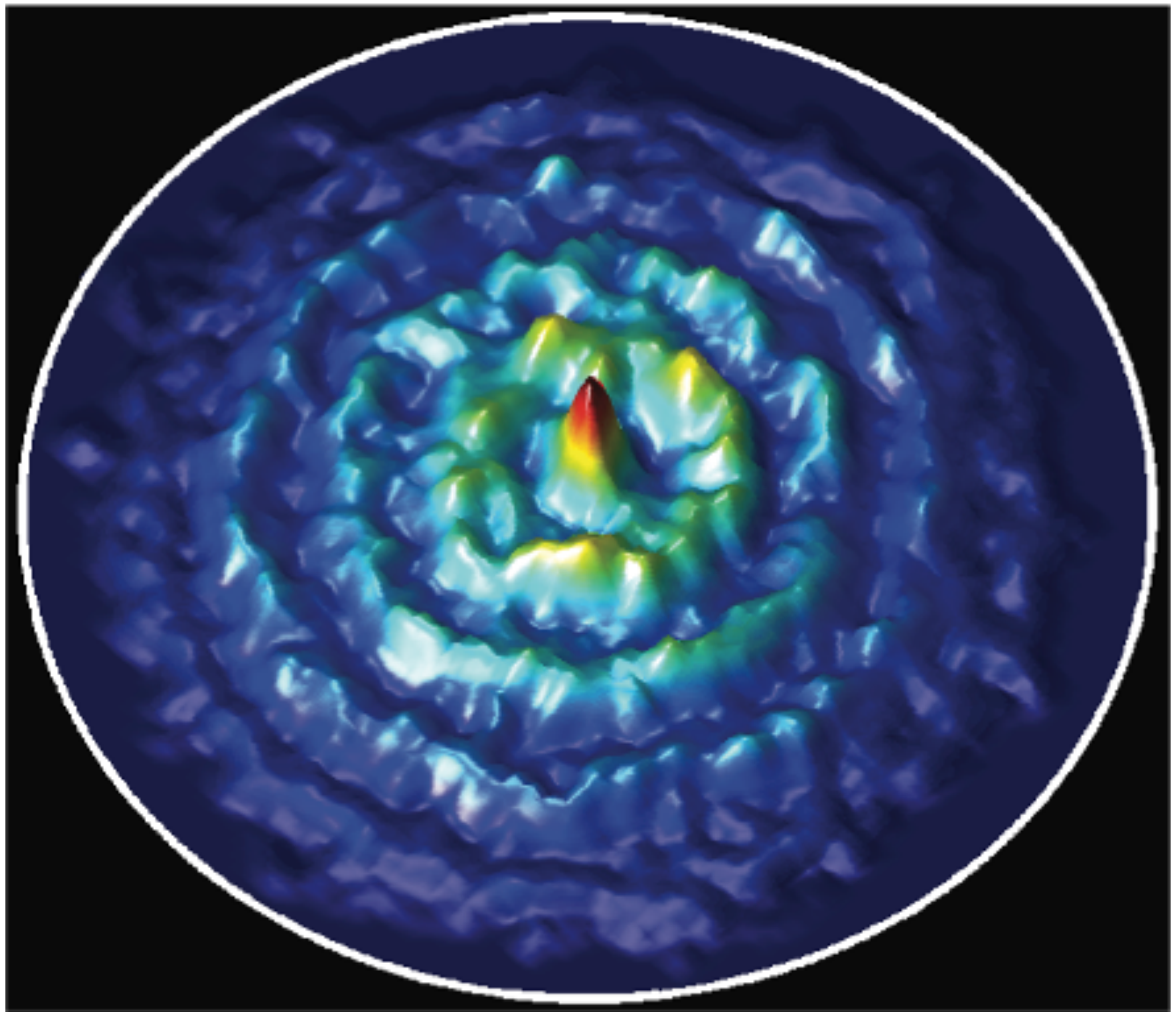
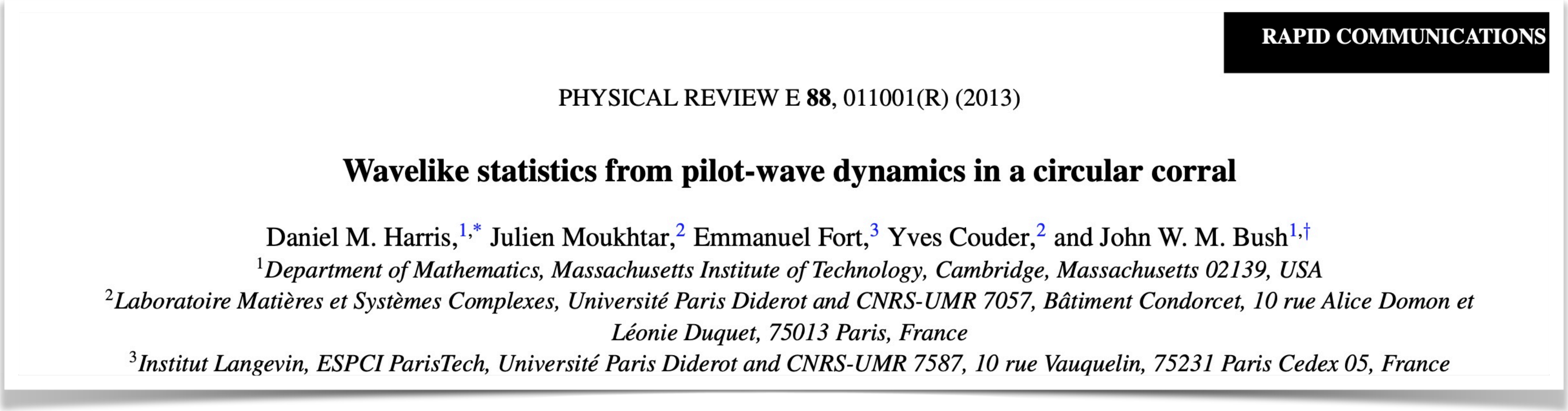
[3] *Institut Langevin, ESPCI ParisTech, Université Paris Diderot and CNRS-UMR 7587, 10 rue Vauquelin, 75231 Paris Cedex 05, France*

Probability distribution



- Emergence of wave-like statistics from chaotic pilot-wave dynamics



RAPID COMMUNICATIONS

PHYSICAL REVIEW E **88**, 011001(R) (2013)

Wavelike statistics from pilot-wave dynamics in a circular corral

Daniel M. Harris,[1,*] Julien Moukhtar,[2] Emmanuel Fort,[3] Yves Couder,[2] and John W. M. Bush[1,†]

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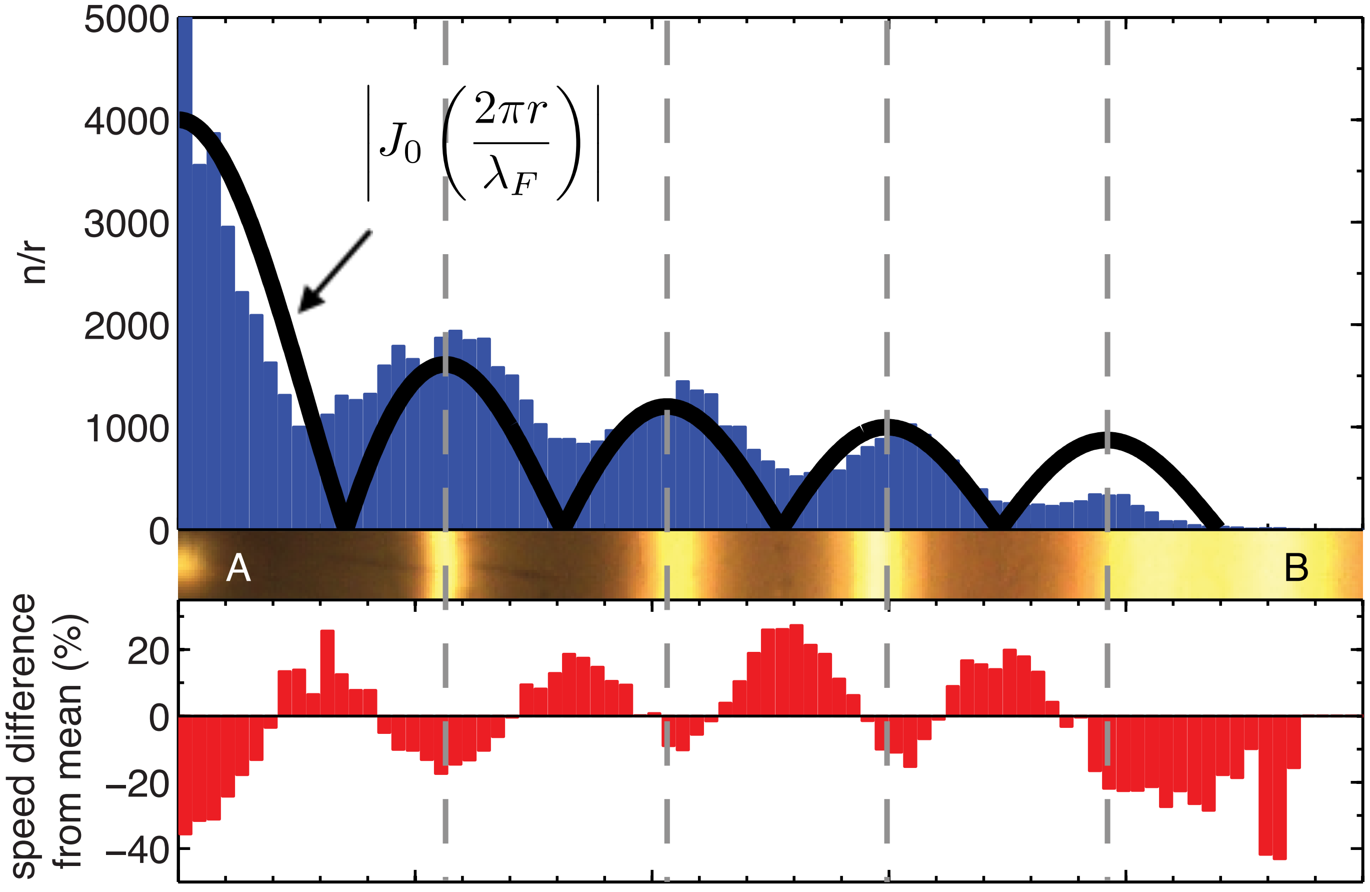
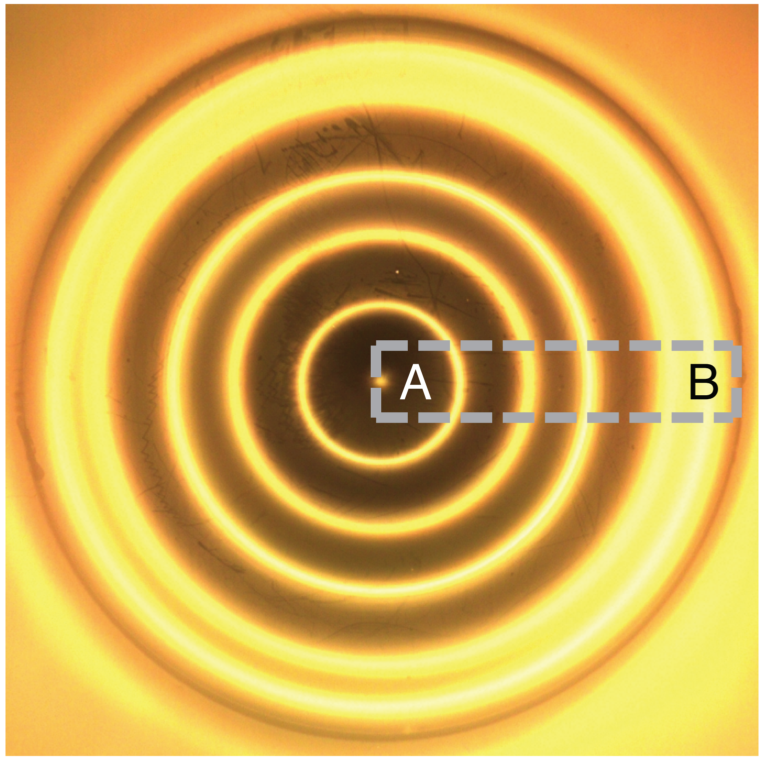
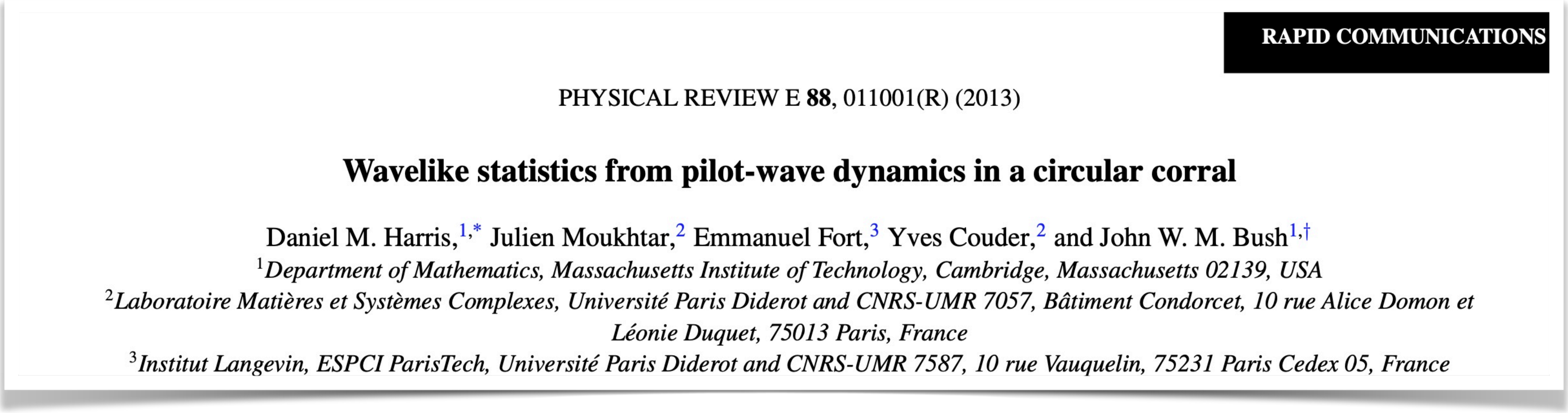




- Statistics are well described by amplitude of most unstable cavity mode



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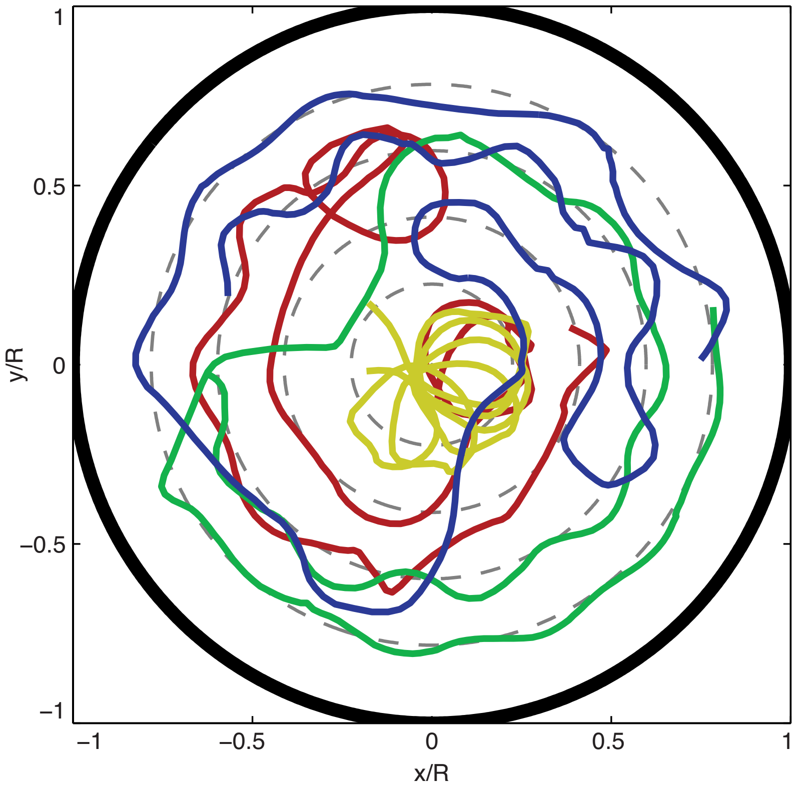
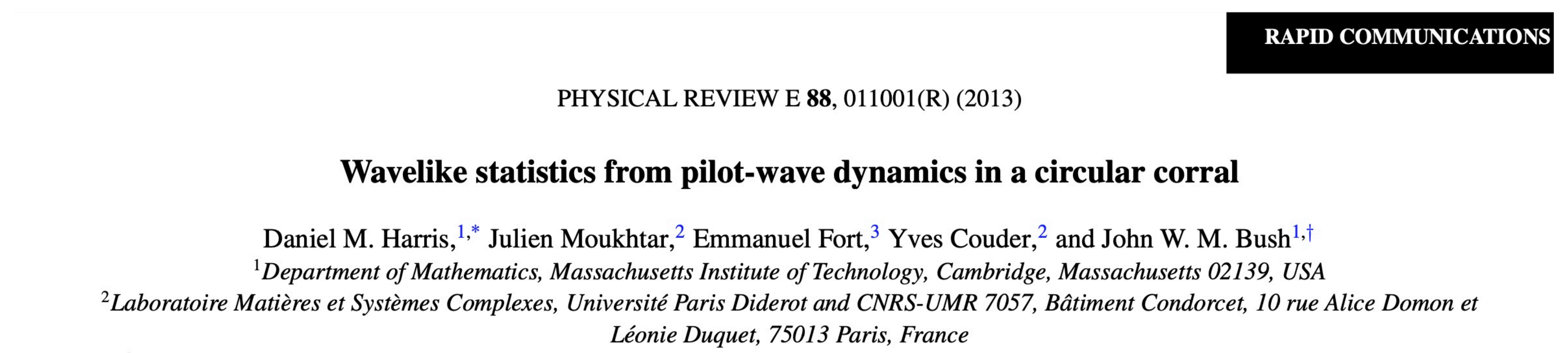
Switching between unstable circular orbits



- Trajectory can be decomposed into arcs along extrema of most unstable cavity eigenmode
- In a simplified 1D model, T. Gilet (2014) demonstrated that the **chaotic switching** between unstable fixed points gives rise to **wave-like statistics**



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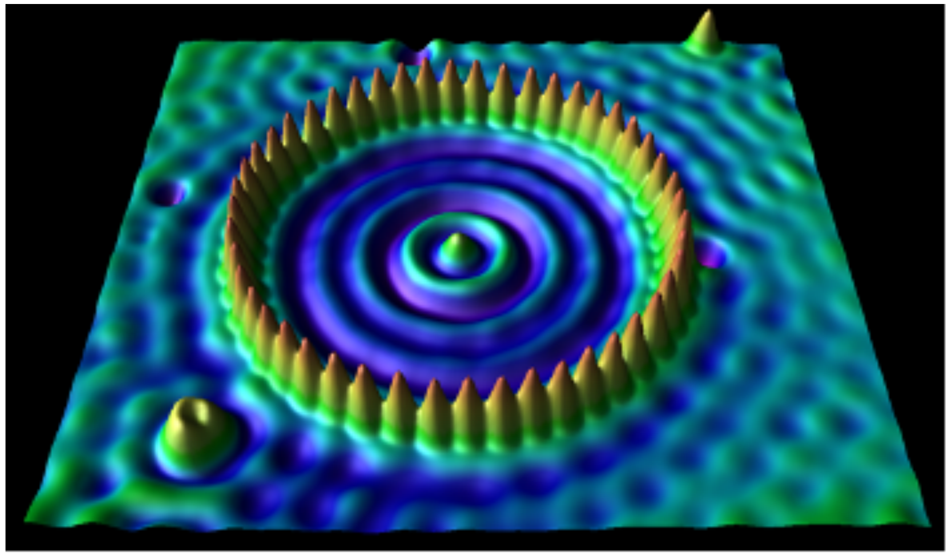
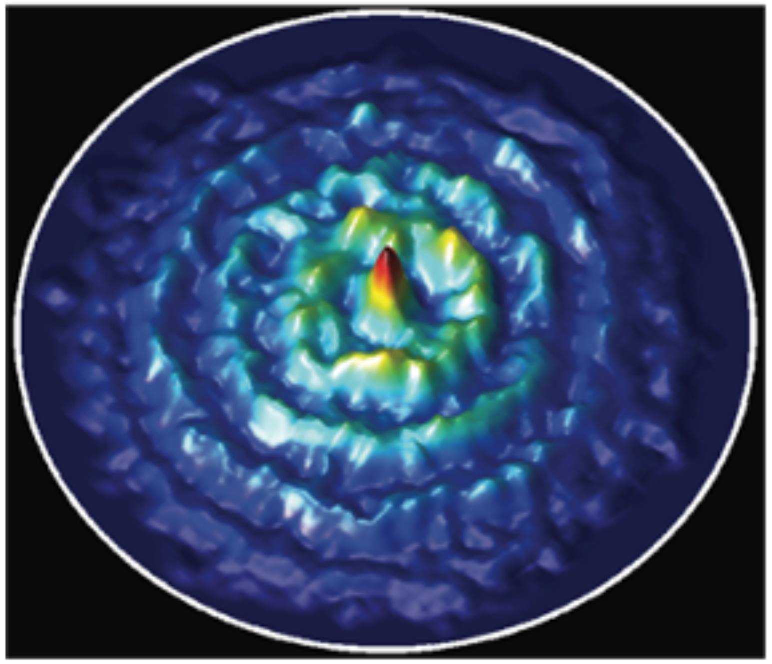
Analogy with quantum corrals



Crommie, Lutz, & Eigler, *Science* (1993)

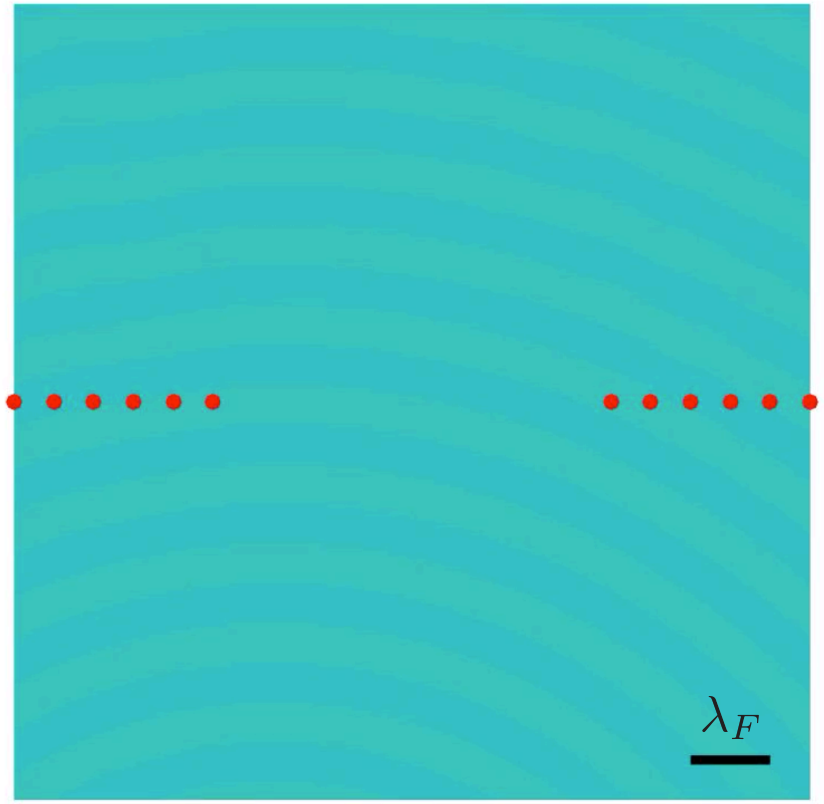


Harris *et al.*, *PRE* (2013)

- In quantum corral, electron statistics are defined by the solution to Schrodinger equation in circular geometry with **de Broglie** wavelength
- In fluid corral, walker statistics are defined by the solution to wave equation in circular geometry with **Faraday** wavelength

Single-particle diffraction with a hydrodynamic pilot-wave model

Physical Review E 111, L033101 (2025)

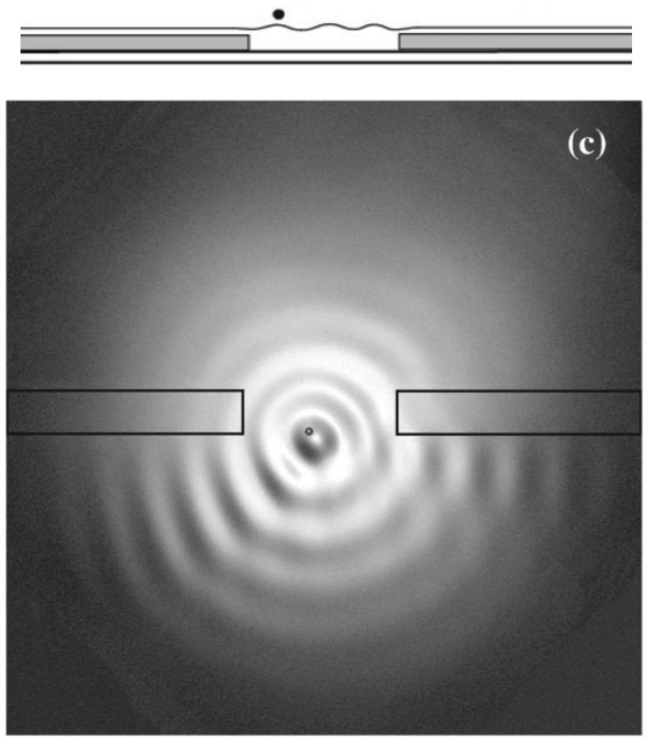


Giuseppe Pucci, *Consiglio Nazionale delle Ricerche - CNR-Nanotec at Università della Calabria.*

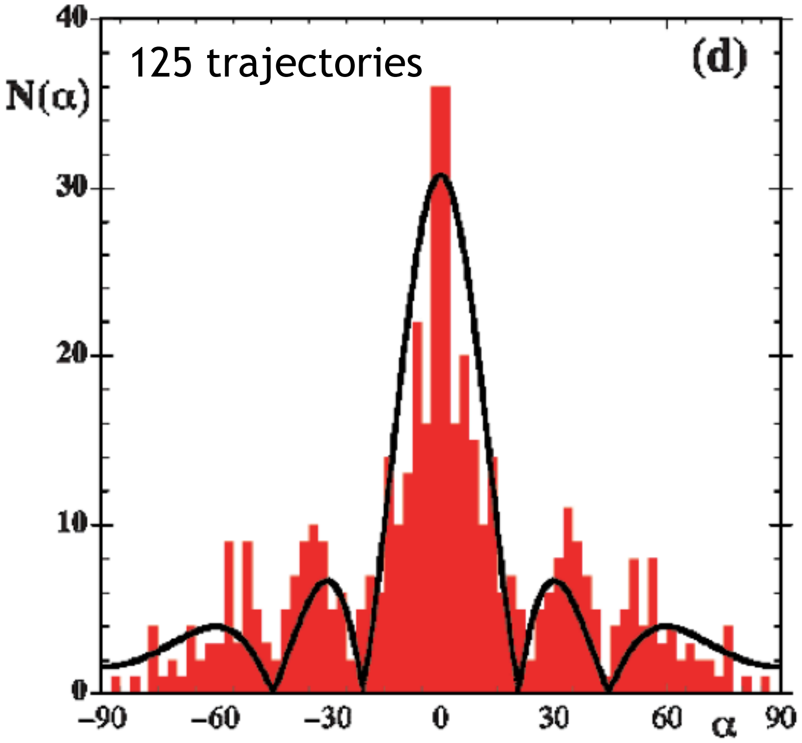
A. Bellaigue, *Institut de Physique de Rennes (France)*
A. Cirimele and **G. Alì**, *Dipartimento di Fisica - Università della Calabria*
A. U. Oza, *New Jersey Institute of Technology (USA).*

What about **diffraction**?

Couder and Fort, *Phys. Rev. Lett.* 2006.



Walking droplets passing through an aperture between two submerged barriers.

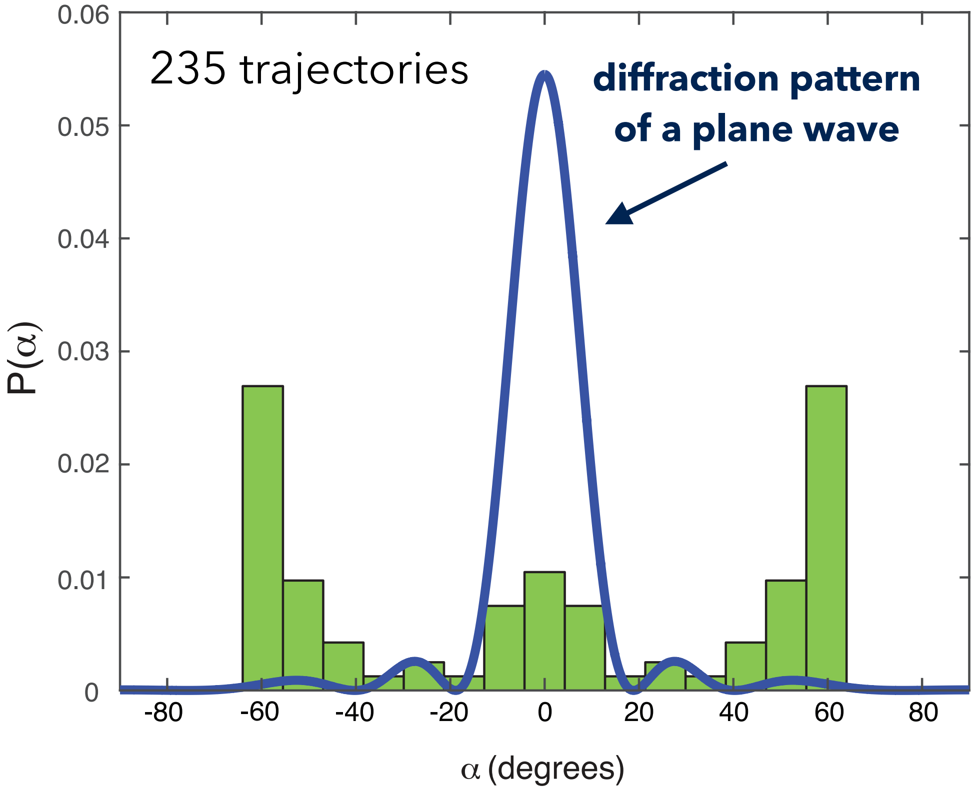


Black curve: Fraunhofer's amplitude.

Revisited by Batelaan *et al.* (2016) with inconclusive results.
Andersen A. *et al.* (2015), Bohr *et al.* (2016) contested Couder & Fort's results on the basis of:

- insufficient data
- their different results
- their questioning the wave interference due to the second slit.

Pucci G. *et al., J. Fluid. Mech.* 2018.



Conclusion: **no quantum-like diffraction** due to **specific hydrodynamic conditions at the barriers**.

Further revisited by Ellegaard and Levinsen, *Phys. Rev. E* 2020 and 2024.

Conclusion: **no quantum-like diffraction**.

use existing models of walking droplets.

Our model

We use the "**stroboscopic model**" of Oza *et al. J. Fluid Mech.* (2013), Couchman et al. *J. Fluid Mech.* (2019), which was benchmarked against experiments reporting **quantization of orbits and angular momentum**.

Equation of motion

$$m\ddot{\mathbf{x}}_p + D\dot{\mathbf{x}}_p = -mg\mathcal{C}\nabla h(\mathbf{x}_p, t)$$

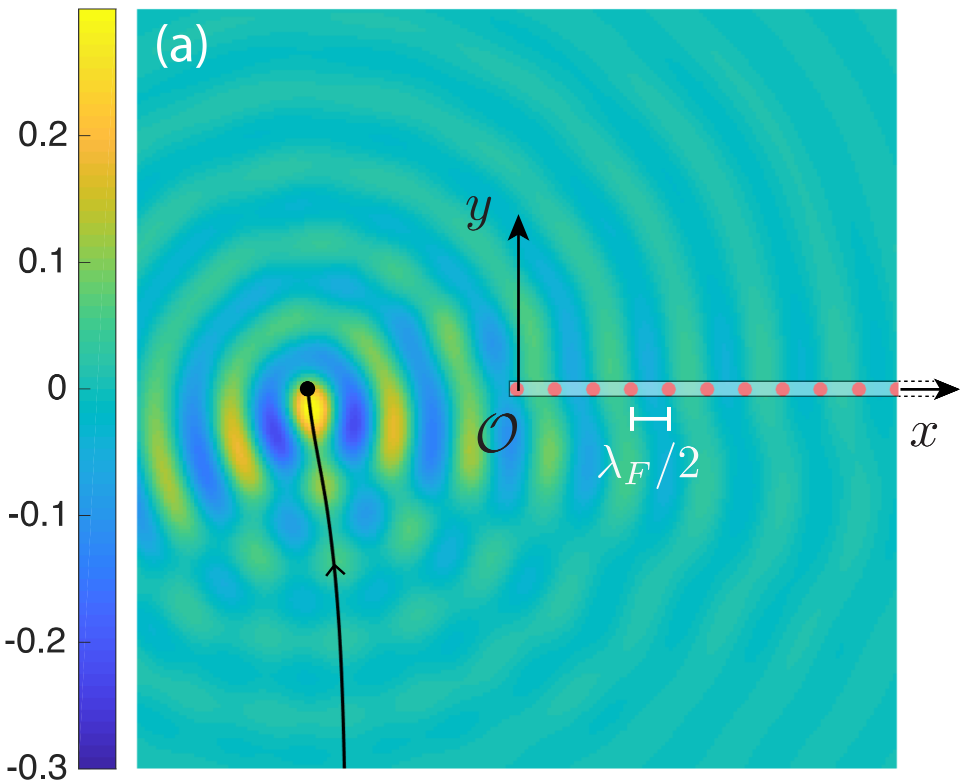
inertia + drag = propulsive force from waves

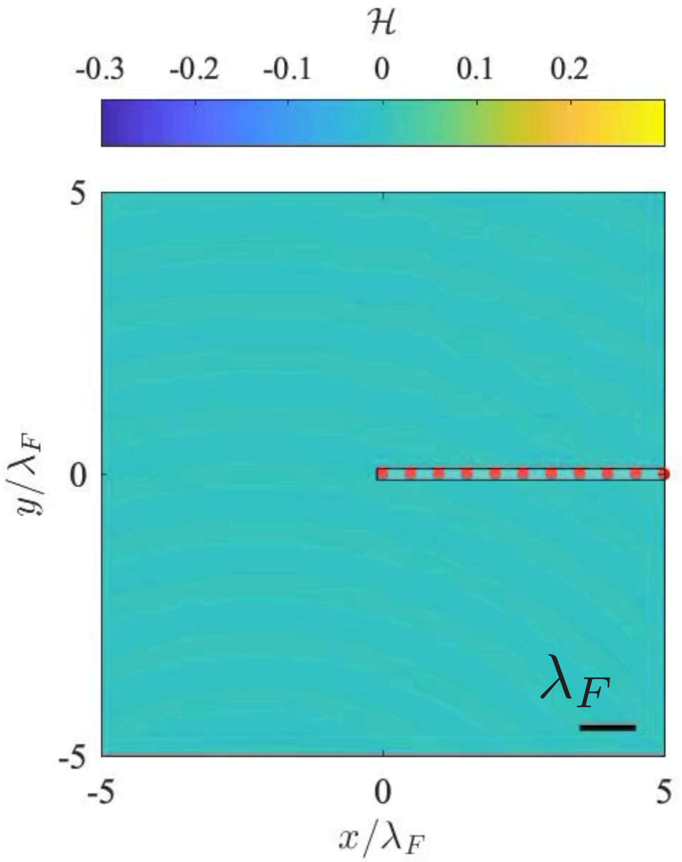
$\mathbf{x}_p$: particle position

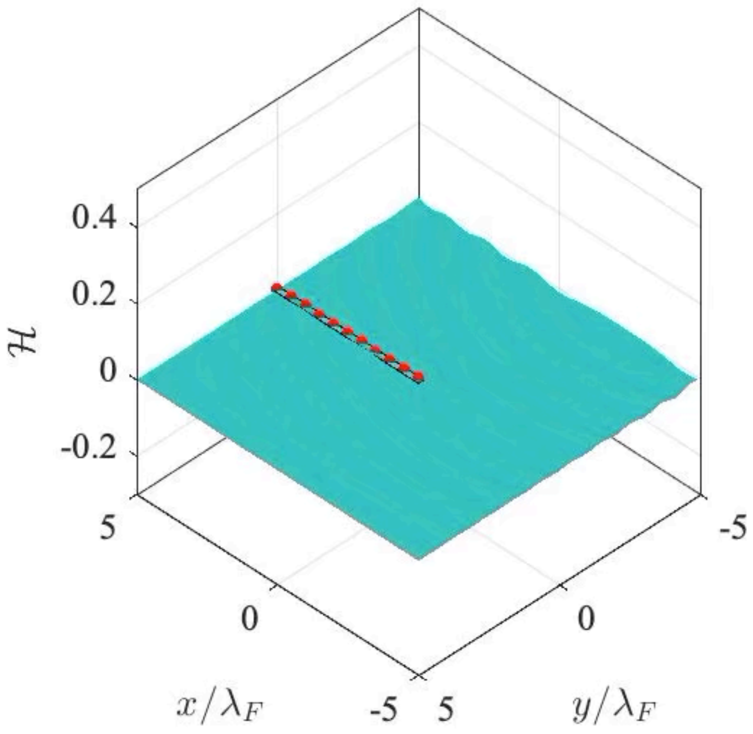
h : wave height

$\mathcal{C}$: average phase of drop impact with respect to the oscillating wave field

- Walls are modelled as series of secondary sources with inter-source spacing $dx = 0.5\lambda_F$ as in Couder & Fort's simulations (2006).
- Source amplitude is chosen in order to have zero amplitude on the sources (perfectly reflecting sources).

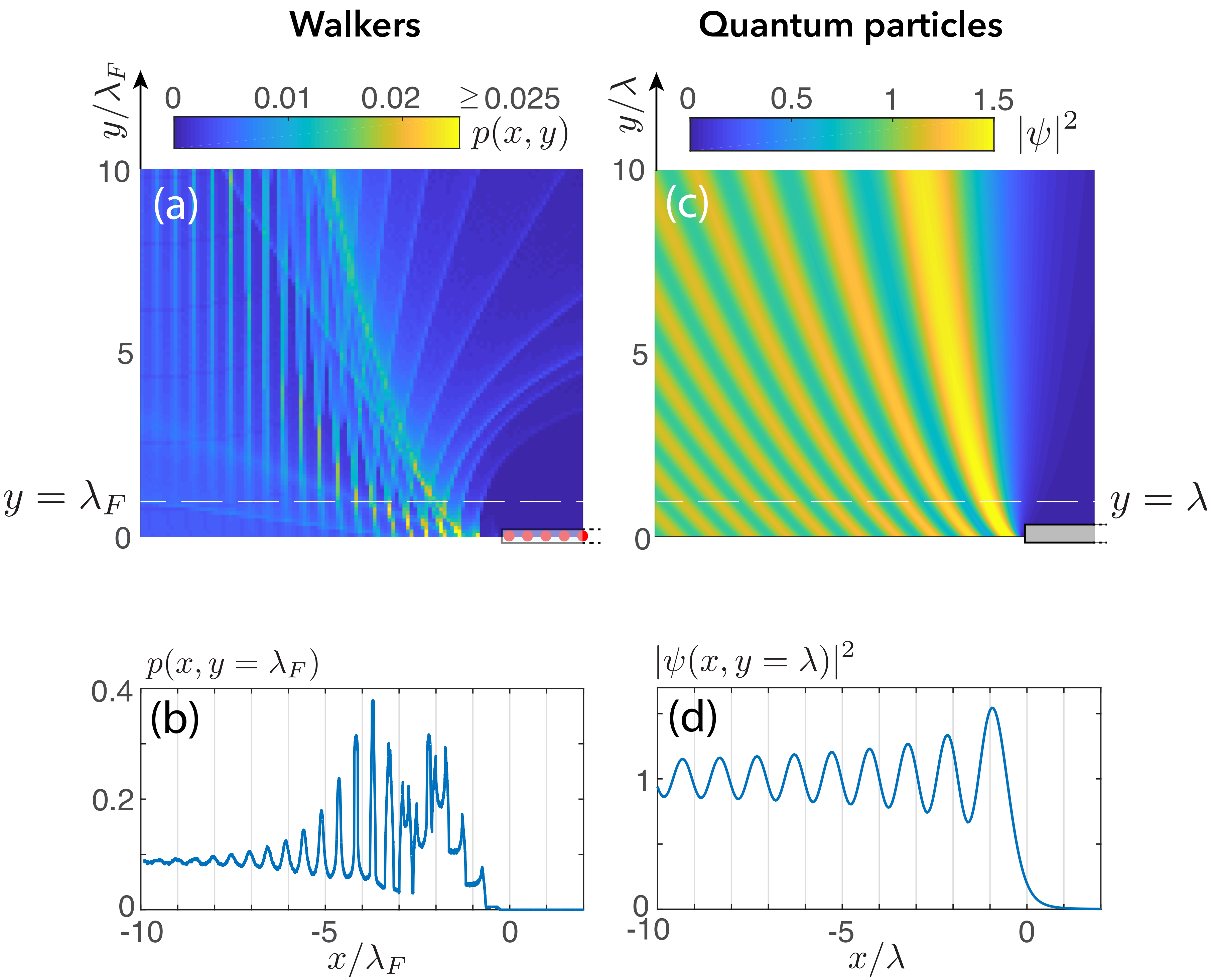






System dissipative –> focus on the **near field** (a few wavelengths away from the barrier)

Diffraction from an edge

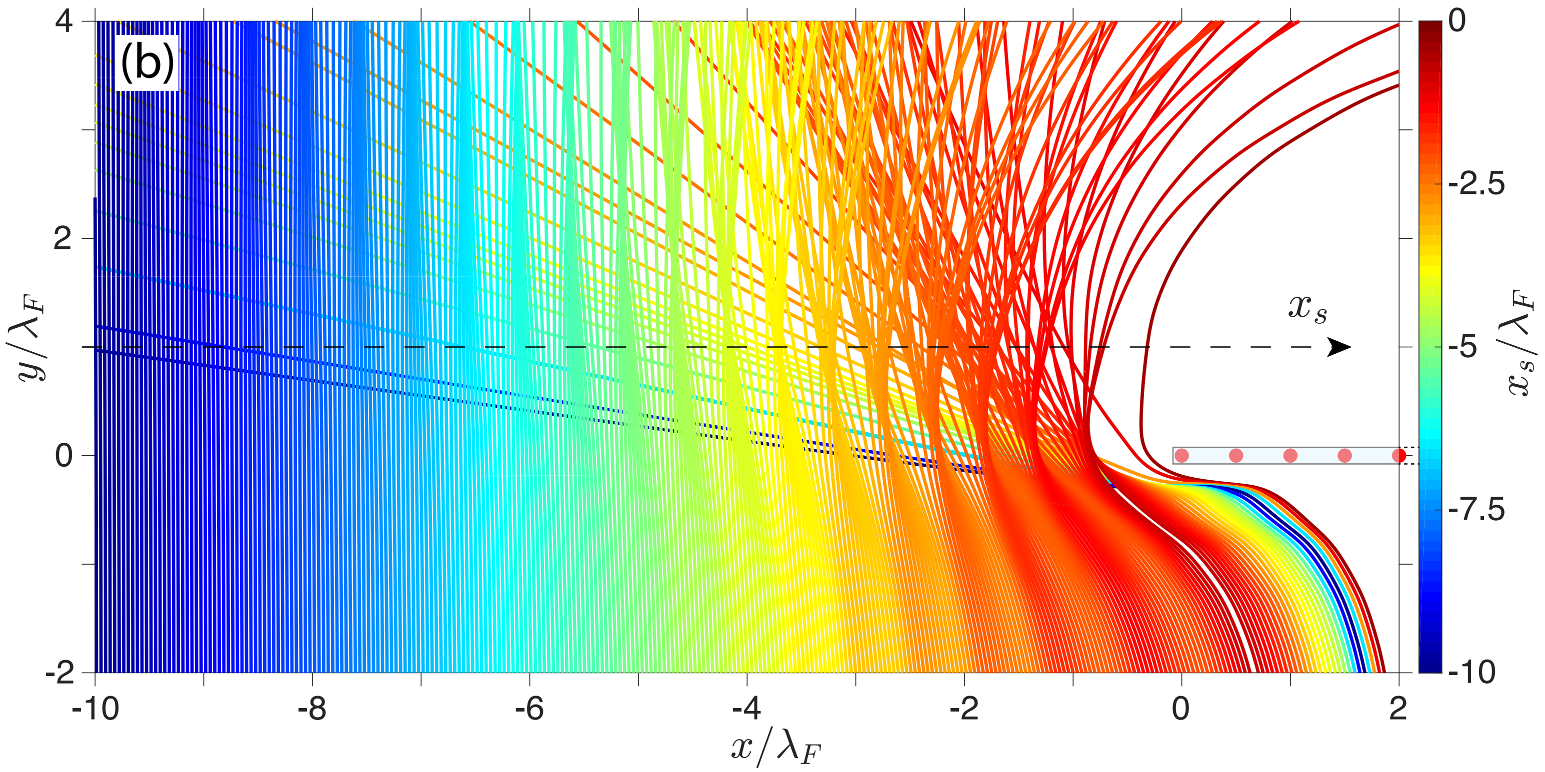


- PDF of **walkers** vs. probability density of **quantum particles**

- Wavelike behavior observed on the PDF of walkers.

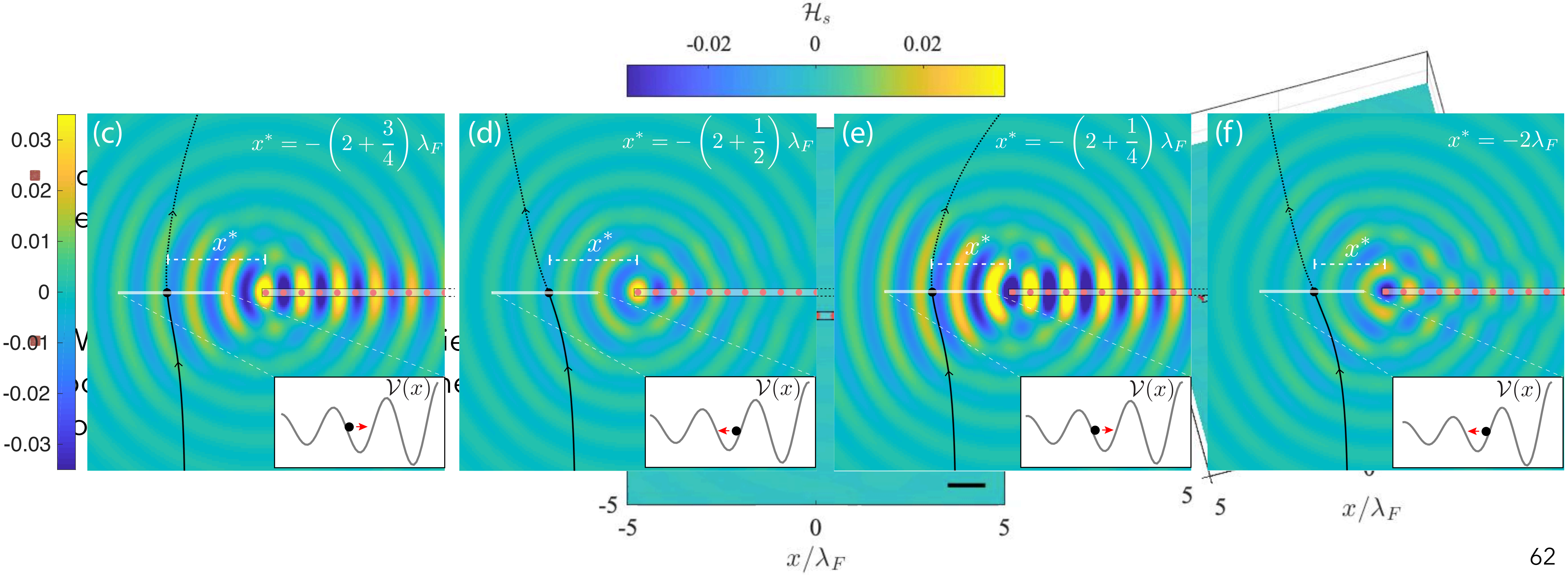
- PDF of walkers oscillate on $\lambda_F/2$ while $|\psi|^2$ oscillates on $\approx \lambda$ (de Broglie wavelength)

Diffraction from an edge: mechanism

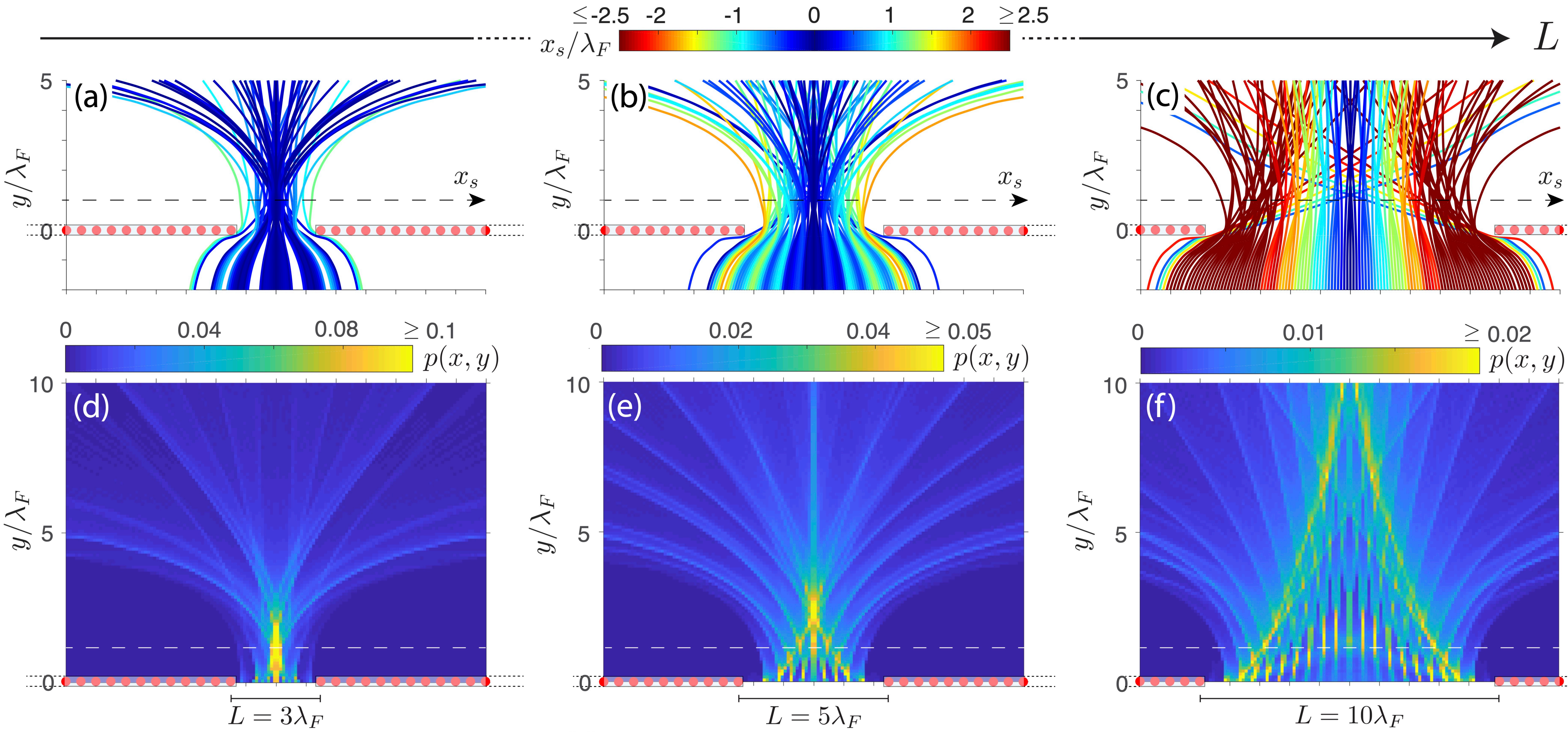


Trajectories color coded according to their position on a screen

λ_F : field wavelength.



Diffraction past a single slit

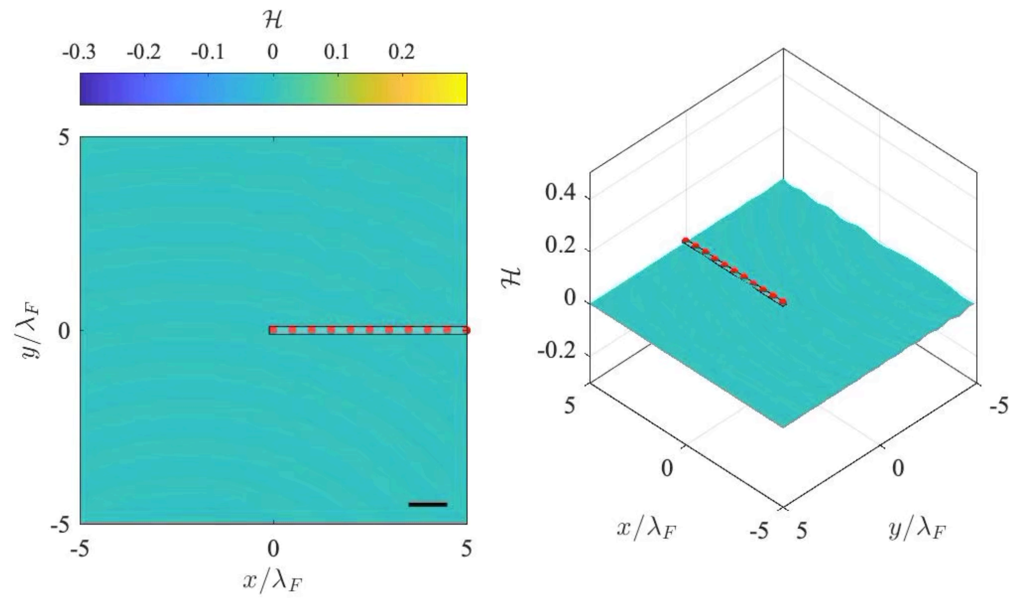


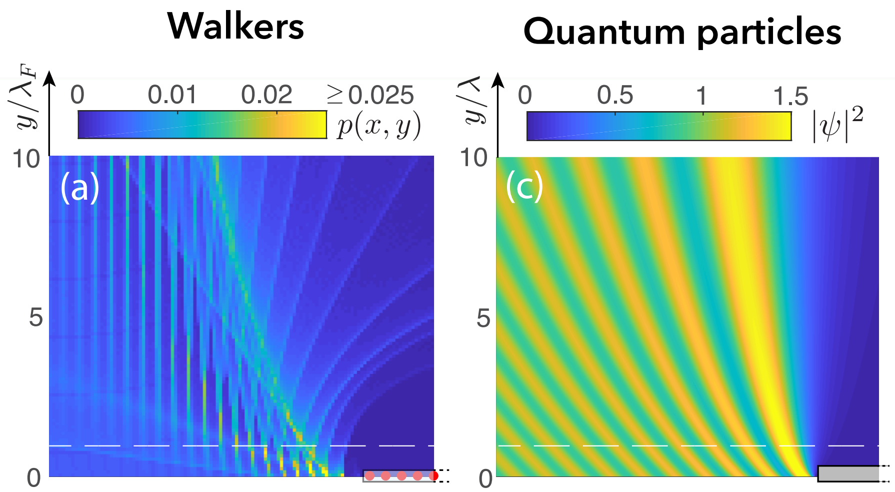
Walkers

Quantum particles

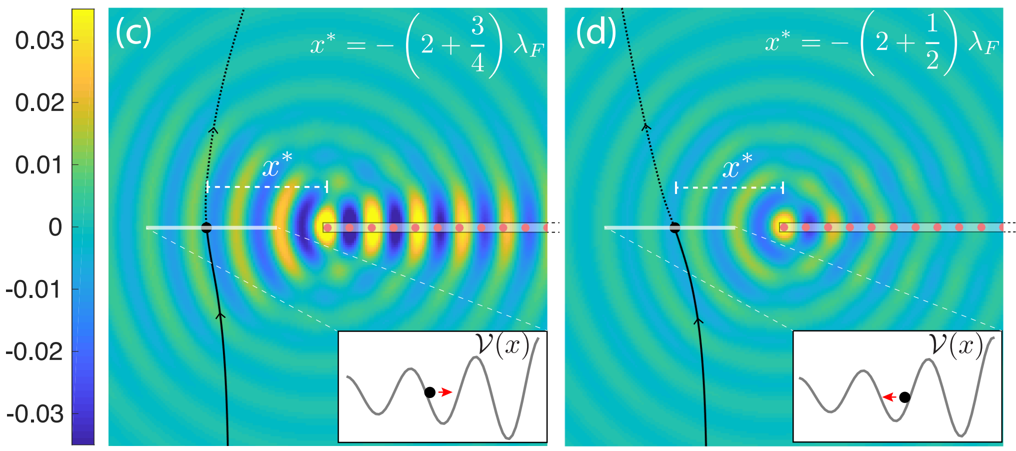
Conclusions on diffraction

- The possibility of obtaining **quantum-like diffraction** with walkers has been elusive until now in **experiments**.
- We have used a hydrodynamic pilot-wave **model** to investigate walker diffraction by linear barriers.
- Since the system is **dissipative** (the walker wave field decays exponentially in space) we focused on the **near field** (not explored in prior studies).
- **We found wavelike particle statistics**, qualitatively similar to quantum particles.
- We have **rationalized single-particle diffraction** in terms of wavefield generated by the barriers, which creates a *transient potential* for the particle.
- These results inform **walker-inspired pilot-wave theories**, which could yield results even closer to quantum mechanics.




Walkers

Quantum particles

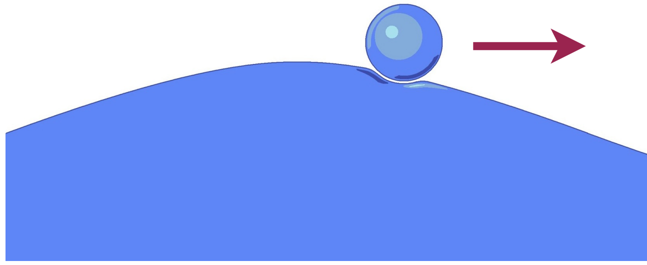


de Broglie's pilot-wave theory: the double-wave solution

"A freely moving body follows a trajectory that is orthogonal to the surfaces of an associated wave guide". (Louis de Broglie (1892-1987))

- Ψ is the probability wave, as prescribed by standard quantum theory
- a **real physical wave** is generated by **internal particle vibration** (Zitterbewegung) at the Compton frequency:

$$\omega_c = \frac{m_0 c^2}{\hbar}$$



- $\Psi^{dB} = |\Psi^{dB}|\, e^{i\phi/\hbar}$ is the **real physical wave** solution of Klein-Gordon equation, responsible for guiding the particle
- it is a **monochromatic standing wave** in the particle frame of reference
- the particle is pushed perpendicular to surfaces of constant phase:

$$\mathbf{p} = m\dot{\mathbf{x}}_{\mathbf{p}} = \nabla\phi = \hbar\mathbf{k} \qquad \text{for a monochromatic wave} \qquad \Psi^{dB} = |\Psi^{dB}| e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$$

- **Harmony of Phases**: the particle oscillates in resonance with its guiding wave

de Broglie, L. *Ondes et mouvements* (1926). / *Non-linear wave mechanics: a causal interpretation* (1960).

de Broglie's pilot-wave theory vs. walkers

- **fast dynamics**: mass oscillations at

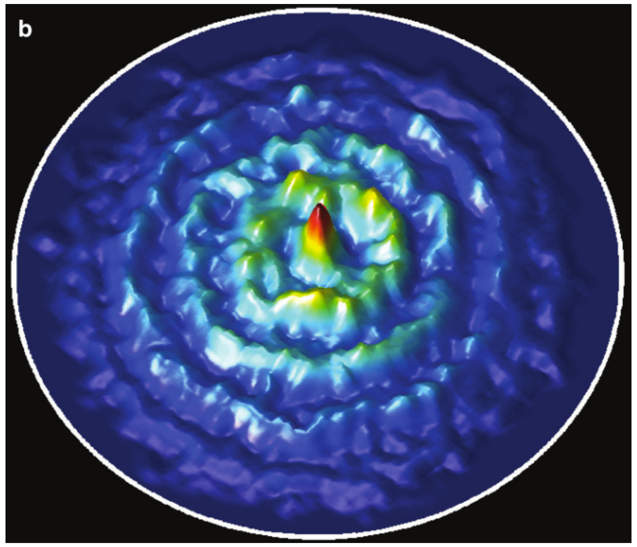
$$\omega_c = \frac{m_0c^2}{\hbar}$$ creates wave field

centered on particle

- **intermediate pilot-wave dynamics:**

particle rides its guiding wave field

- **long-term statistical behavior** described by standard quantum theory



Who currently works in this field